Physics 504: Statistical Mechanics
HOMEWORK SHEET 6

Due 5pm Fri 15 April 2016 in the 504 box.

Please attempt these questions without looking at textbooks, if you can. If you do need to refer to my notes or textbooks, the most effective way to do this is to read the relevant section, and then try the question again without looking at the book/notes.

Question 6–1.

Photons are quantum mechanical bosons with unit spin, 2 helicity states (the quantum analogue of polarisation), whose number is not conserved. In this question, we will study the statistical mechanics of a gas of such particles, and derive the black body radiation spectrum from the "particle" point of view, as opposed to the "wave" point of view presented in lectures.

(a) What is the chemical potential of a gas of photons?
(b) Write down the partition function for a gas of photons with states labelled by momenta $\mathbf{k}$ with energies $E_{\mathbf{k}}$ and occupation numbers $n_{\mathbf{k}}$.
(c) For photons, the dispersion relation is $E_{\mathbf{k}} = \hbar c|\mathbf{k}|$. Hence write down the free energy of a gas of photons in a volume V. You may use the fact that $\int_0^\infty x^2 \log\left(1 - e^{-x}\right) dx = -(2\pi)^4/720$.
(d) Hence calculate the average energy of the gas. You should recover the result obtained in lectures by another method.

Question 6–2.

Surface waves on liquid helium, at angular frequency ω and wavenumber $k = 2\pi/\lambda$ satisfy the following relation: $\omega^2 = [\sigma/\rho]k^3$ where σ is the surface tension of liquid helium and ρ is the density of liquid helium. Suppose that the liquid helium is at temperature T, and has surface area A. Show that the surface energy $U(T)$ — *i.e.* the energy of the surface waves — is given by

$$U(T) = CI\left(\frac{k_B T}{2\pi\hbar}\right)^\alpha$$

where $I = \int_0^\infty dx x^{4/3}/(e^x - 1) \approx 1.68$, and determine the coefficient C and the value of the exponent α.

Question 6–3.

Consider a quantum paramagnet of N magnetic moments with $L = 0$ and $S = 1/2$.

(a) Calculate the contribution to the entropy from the magnetic moments, and sketch the result as a function of $1/x$, where $x = g\mu_B JH/k_B T$. By sketch, I mean figure out what the graph looks like, **not** plotting it on the computer!
(b) Now suppose that this system is at temperature T_1 and in a field H_1, and is thermally isolated from the surroundings. What happens to the temperature as the external field is reduced to zero?
(c) This phenomenon is a powerful experimental technique. What practical limitation does it have? Suggest how the method may be improved.