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# Josephson interference phenomena above $T_c$

Efrat Shimshoni, Paul M. Goldbart, and Nigel Goldenfeld

*Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801-3080 and Beckman Institute for Advanced Science and Technology, 405 North Mathews Avenue, Urbana, Illinois 61801*

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Superconducting fluctuations induce remnants of the Josephson effect above $T_c$. To explore this, we consider a paraconducting cylinder, composed of two thin films connected by tunnel junctions, and threaded by magnetic flux. The electrical conductance includes a component that is an oscillatory function of the flux, having an amplitude proportional to the square of the Josephson coupling, and decaying exponentially with the circumference over a length scale $\xi(T)$. We estimate the typical magnitude of this effect, and contrast it with related mesoscopic effects in normal-metal systems.

At temperatures $T$ above but close to the superconductor–normal-metal transition temperature $T_c$, remnants of superconducting phenomenology can be observed due to the presence of thermally excited superconducting fluctuations.[1] Examples include the enhancement of Landau diamagnetism[2] and electrical conductivity[3–5] by such fluctuations, the coherence of which persists up to lengths of order of the temperature-dependent correlation length $\xi(T)$ and over times of order the zero wave number relaxation time $\tau_0(T)$. Their consequences are more pronounced in effectively lower-dimensional structures, such as films, wires, and grains, in which sample dimensions considerably narrower than $\xi(T)$ are suppressed.

Below $T_c$, one of the striking consequences of superconductivity is the possibility of supercurrents flowing through insulating barriers or weak links, i.e., the Josephson effect.[6,7] The sensitivity of these currents to magnetic and electric fields is a reflection of coherence in the superconducting order parameter. In particular, magnetic flux penetrating a Josephson junction leads to interference effects characterized by the familiar diffraction pattern for the maximum supercurrent as a function of flux.[8] Similar interference effects are exhibited by a SQUID ring threaded by magnetic flux, for which the diffraction pattern is replaced by a two-slit interference pattern.[9] Furthermore, chemical potential differences cause supercurrents which oscillate in time.[10,1]

As demonstrated by Kulik,[11,12] superconducting fluctuations also lead to remnants of the Josephson effect just above $T_c$. Kulik showed that, due to the weak coupling between fluctuations on either side of a junction, the frequency-dependent conductivity has a resonance with width of order $\tau_0(T)^{-1}$ at the Josephson frequency $\omega_J = 2eV_J/\hbar$, where $V_J$ is the potential difference across the junction. This enhancement of the conductivity is a manifestation of the long *temporal* coherence of the fluctuations near $T_c$.

The purpose of the present paper is to explore the remnants, above $T_c$, of the Josephson effect, associated with the *spatial* coherence of superconducting fluctuations, i.e., the remnants of SQUID phenomenology. We focus on the flux dependence of the electrical conductivity as a probe of the spatial extent of fluctuations.[13] To do this, we choose to consider a multiply connected paraconducting two-junction SQUID-like structure, threaded by a flux. Then, sensitivity to this flux is expected, provided that coherent multiply connected fluctuations can arise with non-negligible probability. We note that the related problem of the flux sensitivity of a perfect thin-walled paraconducting cylinder has been addressed by Kulik and Mal'chuzhenko.[5]

We therefore consider a system composed of two identical hemicylindrical thin films of a paraconductor, connected by two identical tunnel junctions so as to form a cylinder, as depicted in Fig. 1. We study the fluctuation contribution to the conductivity of the system, and show that it contains a component which varies periodically

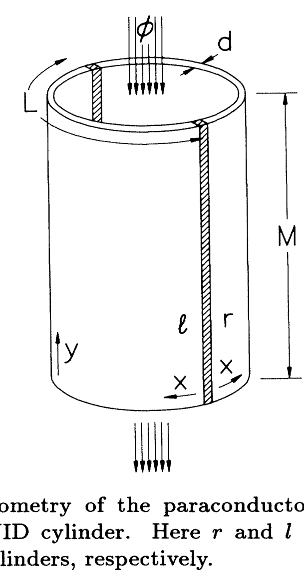


FIG. 1. Geometry of the paraconductor-insulator-paraconductor SQUID cylinder. Here $r$ and $l$ denote the right and left hemicylinders, respectively.

 

with the flux threading the cylinder. The amplitude of this component is attenuated by a factor which is proportional to the square of the Josephson coupling constant, and decays exponentially with the circumference of the cylinder, over a length scale $\xi(T)$. Thus, we expect such a flux dependence to be observable in tubular structures of internal circumference not much greater than $\xi(T)$.

Consider a cylindrical system of circumference $2L$, height $M \gg L$, and thickness $d \ll \xi(T)$, so that the system is effectively two dimensional. The cylinder is threaded by a time-independent magnetic flux $\phi$. As we shall be concerned with temperatures $T$ that are close to but greater than the superconducting transition temperature $T_c$, we describe the system by a quadratic Ginzburg-Landau (GL) free energy $F$,[14,15] given in terms of the superconducting order parameter $\psi$ by

$$F = F^+ + F^- + F^J, \tag{1a}$$

$$F^s = \alpha(T) d \int_0^L dx \int_0^M dy \{ |\psi^s(\mathbf{r})|^2 + \xi(T)^2 \left| [\boldsymbol{\nabla} - 2\pi i s(\phi/2L\phi_0)\mathbf{e}_x] \psi^s(\mathbf{r}) \right|^2 \}, \tag{1b}$$

$$F^J = -Jd \int_0^M dy \{ \psi^+(0,y)^* \psi^-(0,y) + \psi^+(0,y)\psi^-(0,y)^* + \psi^+(L,y)^*\psi^-(L,y) + \psi^+(L,y)\psi^-(L,y)^* \}, \tag{1c}$$

where the superscript $s = +(-)$ denotes the left (right) hemicylinder of Fig. 1, and $F^s$ and $F^J$ are, respectively, the free energies of the uncoupled paraconductors and the junctions. The GL parameter $\alpha(T) = (T/T_c - 1)\alpha'$ represents the usual temperature-dependent condensation energy, the correlation length is given by $\xi(T)^2 = \hbar^2/2m\alpha(T)$, $m$ is the effective mass of the Cooper pair, $\phi_0 = h/2e$ is the superconducting flux quantum, $J$ is the coupling constant associated with each tunnel junction, $\mathbf{e}_\mu$ denotes the unit vector in the $\mu$ direction, and the coordinate system is specified in Fig. 1. We assume that there is a negligible amount of magnetic flux actually passing through the sample.

In the experiment we have in mind, leads are attached to the upper and lower circular edges of the cylinder in Fig. 1. An infinitesimal transport current $I$ is passed between the leads, and associated with this current is a voltage difference $V$. Our aim is to establish the superconducting fluctuation contribution to the conductance $G \equiv I/V$ associated with this setup. To do this, we construct the current

$$I(\omega) = d \sum_s \int_0^L dx\, j^s_{t\,y}(\mathbf{r},\omega) \tag{2}$$

from the transport current density $\mathbf{j}^s_t$, given by Ohm's law

$$j^s_{t\,\mu}(\mathbf{r},\omega) = d \sum_{s'} \sum_{\mu'} \int_0^L dx' \int_0^M dy' \sigma^{s,s'}_{\mu,\mu'}(\mathbf{r},\mathbf{r}',\omega)\, E^{s'}_{\mu'}(\mathbf{r}',\omega), \tag{3}$$

where $\sigma^{s,s'}_{\mu,\mu'}$ is the electrical conductivity, and $\mathbf{E}^s(\mathbf{r},\omega) = M^{-1}V(\omega)\mathbf{e}_y$ is the frequency-dependent electric field corresponding to the envisaged experiment.

To compute $\sigma^{s,s'}_{f\,\mu,\mu'}$, the contribution to the conductivity due to the equilibrium superconducting fluctuations, we use the Kubo formula

$$\sigma^{s,s'}_{f\,\mu,\mu'}(\mathbf{r},\mathbf{r}',\omega) = \frac{1}{k_B T} \int_0^\infty dt\, \langle j^s_\mu(\mathbf{r},0)\, j^{s'}_{\mu'}(\mathbf{r}',t) \rangle \cos\omega t\,, \tag{4}$$

where $\mathbf{j}^s$ is the fluctuating supercurrent, given by

$$\mathbf{j}^s(\mathbf{r},t) = \frac{2e\hbar}{m} \mathrm{Im}\, \{ \psi^s(\mathbf{r},t)^* [\boldsymbol{\nabla} - is(\pi\phi/L\phi_0)\mathbf{e}_x] \psi^s(\mathbf{r},t) \}, \tag{5}$$

in which $-e$ is the electronic charge. We assume that the width of the junction is vanishingly small, compared with $L$, so that a negligible current is carried along the junction itself.

We obtain the required current-current correlator by using the stochastic time-dependent (TD) GL equation for the superconducting order parameter (in the absence of an electric field),

$$\tau_0 \frac{\partial}{\partial t} \psi^s(\mathbf{r},t) = -\frac{1}{\alpha} \frac{\delta F}{\delta \psi^s(\mathbf{r})^*} \Big|_{\psi(\mathbf{r},t)} + \zeta^s(\mathbf{r},t), \tag{6}$$

where $\tau_0 \equiv \pi\hbar\beta/8(1 - T_c/T)$ is the zero wave number relaxation rate, $\beta \equiv 1/k_B T$, and $\zeta^s(\mathbf{r})$ is a random Gaussian noise with mean zero and covariance

$$\langle \zeta^s(\mathbf{r},t)\, \zeta^{s'}(\mathbf{r}',t') \rangle = \frac{2\tau_0}{\alpha\beta}\, \delta(\mathbf{r}-\mathbf{r}')\, \delta(t-t')\, \delta^{s,s'}, \tag{7}$$

chosen to satisfy the fluctuation-dissipation theorem.[16]

It will prove convenient to work in a basis in which the free energy of the uncoupled (i.e., $J=0$) paraconductors is diagonal. To this end, we introduce the complete basis of functions[17]

$$u^s_{kq}(x,y) = \frac{p_k}{\sqrt{dLM}} \exp(is\pi\phi x/\phi_0 L)\, \cos(kx)\, \exp(iqy)\,, \tag{8}$$

orthonormal in their subscripts, where $Mq/2\pi$ ranges over all integers, $Lk/\pi = 0,1,2,\ldots$, and $p_k \equiv 1$ ($p_k \equiv \sqrt{2}$) for $k = 0$ ($k \neq 0$). Expanding the order parameter in this basis,

$$\psi^s(\mathbf{r},t) = \sum_{kq} \psi^s_{kq}(t)\, u^s_{kq}(x,y) \tag{9}$$

(and similarly expanding the noise term), and using orthonormality, the TDGL becomes

$$\tau_0 \partial_t \boldsymbol{\psi}(t) = -\boldsymbol{\Lambda} \cdot \boldsymbol{\psi}(t) + \boldsymbol{\zeta}(t)\,, \tag{10}$$

where the vectors $\boldsymbol{\psi}$ and $\boldsymbol{\zeta}$ have components $\psi^s_{kq}$ and $\zeta^s_{kq}$, and the matrix $\boldsymbol{\Lambda}$ has components $\Lambda^{s,s'}_{kq,k'q'}$, given by

$$\begin{aligned} &f_{kq}\delta_{qq'}\,\delta_{kk'} \quad \text{for } s = s'\,,\\ &\tfrac{-J}{\alpha L}\delta_{qq'}p_k p_{k'}\left\{1+\cos(kL)\,\cos(k'L)\,e^{i(s-s')\pi\phi/\phi_0}\right\}\\ &\qquad\qquad\qquad\qquad\qquad\qquad \text{for } s \neq s'\,, \end{aligned}$$

in which $f_{kq} = 1 + \xi(T)^2(k^2+q^2)$.

Following the standard approach,[16] we obtain the Fourier transform of the stationary correlation function

$$\begin{aligned} S^{s,s'}_{kq,k'q'}(\omega) &\equiv \int_{-\infty}^{\infty} dt\, e^{-i\omega t}\langle \psi^s_{kq}(t)\psi^{s'}_{k'q'}(0)^*\rangle \\ &= \frac{2\tau_0}{\alpha\beta}\left(\boldsymbol{\Lambda}+i\omega\tau_0\mathbf{I}\right)^{-1}\left(\boldsymbol{\Lambda}^\dagger - i\omega\tau_0\mathbf{I}\right)^{-1}\Big|^{s,s'}_{kq,k'q'}, \end{aligned} \tag{11}$$

where $\mathbf{I}$ is the appropriate identity matrix. Employing Eqs. (2)–(5), we find the fluctuation contribution to the conductance

$$G_f(\omega) = \left(\frac{e\hbar}{mM}\right)^2 \frac{1}{\pi k_B T} \sum_{ss'qq'kk'} qq' \left(S^{s,s'}_{kq,k'q'} \circ S^{s',s}_{k'q',kq}\right)(\omega)\,, \tag{12}$$

where $\circ$ denotes Fourier convolution.

Using Eq. (11), we evaluate $S^{s,s'}_{kq,k'q'}(\omega)$ perturbatively, to second order in $J/\alpha L$, and insert the result into Eq. (12). We then evaluate the remaining summations at $\omega = 0$, retaining terms to leading order in $L/\xi(T)$, thus obtaining the dc fluctuation conductance $G_f(0)$,

$$G_f(0) = G_{\mathrm{AL}}\left\{1 + \frac{15\pi}{16}\left(\frac{J}{\alpha L}\right)^2 \frac{L}{\xi(T)}\left[1 + \frac{32}{15\sqrt{\pi}}\left(\frac{L}{\xi(T)}\right)^{3/2} e^{-2L/\xi(T)}\cos(2\pi\phi/\phi_0)\right]\right\}, \tag{13}$$

which we have expressed in terms of the conductance $G_{\mathrm{AL}} = (2Ld/M)\sigma_{\mathrm{AL}}$, with $\sigma_{\mathrm{AL}} \equiv e^2/16d\hbar(1-T_c/T)$ being the usual Aslamazov-Larkin contribution to the conductivity of an effectively two-dimensional paraconducting film.[4] Equation (13) is the principal result of this paper, demonstrating the existence of interference phenomena due to the coherence of superconducting fluctuations over a length scale $\xi(T)$. Higher-order perturbative contributions in $J$ lead to higher harmonics in the flux dependence of the conductance, but with additional exponential attenuation in $2L/\xi(T)$. Unlike the case of the homogeneous cylinder (i.e., uninterrupted by junctions), even for $L \lesssim \xi(T)$ the essential contribution to the flux dependence arises at the fundamental frequency, higher harmonics being suppressed. It should be noted that the total conductance $G$ does of course contain a normal contribution $G_n$, in addition to $G_f$.

We anticipate qualitatively similar flux sensitivity for the conductance associated with an alternative experiment, in which the current is passed perpendicular to the cylinder axis, necessarily passing through the junctions. In this case, the normal contribution to the conductance is due to single-electron tunneling through the junctions, and is therefore suppressed with respect to the axial normal conductance (i.e., $G_n$) by a factor proportional to $J$. Hence, the flux-dependent term, being $\mathcal{O}(J^2)$, will be relatively more pronounced. However, a slightly more refined theoretical treatment seems appropriate for this case, owing to the necessity of considering inhomogeneous electric fields.[18]

For the sake of illustration we give a rough estimate of the magnitude of the flux-dependent part of the conductance relative to the Aslamazov-Larkin part. To do this, we take $L \sim \xi(T)$, where $\xi(0) \sim 1000$ Å and $(T/T_c - 1) \sim 10^{-2}$, $\alpha(0) \sim 10^{-1}\,\mu$eV and $J \sim 10^{-8}\,\mu$eV cm, corresponding to the geometrical parameters $d \sim 10^{-2}\,\mu$m, $L \sim 1\,\mu$m and $M \sim 10\,\mu$m, and the normal state junction resistance $R_n \sim 50\ \Omega$. Then $[G_f(0) - G_{\mathrm{AL}}]/G_{\mathrm{AL}} \sim 10^{-2}\cos(2\pi\phi/\phi_0)$.

Related phenomena may be anticipated for a single junction above $T_c$ in the presence of a magnetic field in the junction plane. However, in contrast with the situation below $T_c$, imperfect diamagnetism above $T_c$ is not sufficient to confine the magnetic field to the junction region. Thus, the flux-dependent contribution to the phase of the junction tunneling amplitude is not unique (i.e., is dependent on the initial and final position). Therefore, the flux-dependent contribution to the conductance (and all macroscopic observables) is suppressed.[19]

There is an additional contribution to the paraconductivity, known as the Maki-Thompson contribution.[1] This contribution increases, relative to the Aslamazov-Larkin contribution, the lower the dimension. The present theory does not account for the Maki-Thompson contribution. Thus, the quantitative estimate of the effects predicted here are expected to be more accurate for higher-dimensional structures (i.e., cylinders) than for lower-dimensional ones (i.e., rings). We do, however, expect that the effects reported here would occur in rings as well as cylinders, in fact being more pronounced for rings.

It is worthwhile to contrast the present interference phenomena with those exhibited by *normal* mesoscopic rings and cylinders threaded by magnetic flux,[20] a juxtaposition which has received considerable attention recently.[21] In such structures, single-particle phase coherence leads to flux sensitivity, e.g., of the conductance, provided the circumference does not greatly exceed the dephasing length $L_\phi$; in the paraconducting environment $\xi(T)$ plays an equivalent role. However, the temperature dependence of the two origins of flux-dependence are quite distinct. Thus, in order to deconvolute paraconducting from single-particle mesoscopic effects, $L_\phi$ should be much smaller than $\xi(T)$, over as wide a range of temperatures near $T_c$ as possible. This range is larger at higher temperatures, where $L_\phi$ is small, e.g., due to electron-phonon scattering. This *desideratum* is not realized in aluminium, but may be achievable in a higher $T_c$ superconductor. Moreover, in rings the lack of self-averaging leads to single-electron conductance oscillations with period $2\phi_0$ $(= h/e)$. In contrast, oscillations due to paraconductivity occur with period $\phi_0$.

Finally, we note that in effectively one-dimensional ring structures the fluctuation correlations induced by a Josephson junction are analogous to single-particle scattering by a localized impurity potential. However, the analogy ceases for long cylinders. In the normal cylinder the impurity configuration varies randomly along the length,[22] whereas in the paraconducting cylinder the location of the junction remains fixed. Consequently, in contrast with the case of the normal cylinder, the contribution to the flux dependence of the conductance with (fundamental) period $\phi_0$ does not average to zero.

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[17] The following boundary conditions are satisfied by this set of functions: (i) $\big(-i\partial_x - s\pi\phi/\phi_0 L\big)u^s_{n,q}(x,y)|_{x=0,L} = 0$; and (ii) $u^s_{n,q}(x,y)|_{y=0} = u^s_{n,q}(x,y)|_{y=M}$.

[18] The straightforward application of the present formalism to this alternative geometry leads to $\nabla\cdot\mathbf{j}^s_t \neq 0$, indicating the explicit need to solve Maxwell's equations simultaneously with the TDGL.

[19] The cylindrical geometry considered here evades such cancellation because of the decomposition of the Feynman paths into discrete winding number sectors.

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