Homework assignment 4 for Phys498 Biological Information and Complexity
submitted by Anoush Aghajani-Talesh on dec. 10, 2001

Dynamics of Urban Sprawl

In this essay a model for the spatial development of a contemporary city and its implemantation on a computer is presented. It is based on work done by Batty et al [1]. The spatial dynamics of Urban growth is characterized by three phenomena: 1. The decline of the core of the city, which implies that certain central areas become less attractive, or are even abandoned by business and residents. 2. The emergence of edge cities, where new business and industry is established. 3. The rapid suburbanisation of the peripherie of the city, which was made possible by progress in transport and communication technologies lowering costs of the spread of consumption and production. Attempts to control and to manage the growth process proved inspite of manifold efforts to be of limited and temporarilily succes. This is also due to the lack of understanding of urban dynamics, which is a very complex process governed by manifold social and economical influences. A model that would include all important factors related to preferences for work and living, real estate market and prizing, transportation and traffic is likely to fail due to excessive realism. Instead the models discussed below abstract the urbanisation to a high degree and focus on the geometric properties of the urban sprawl. Despite its generality it is supposed to be able to explain and to simulate different types of devolopments by only few parameters.

We start with a simple model of the dynamics that is commenly used to describe the spread of a disease through infection, in order to let the general ideas become more apparent. However, better results can be obtained with a more refined approach. Its implementation on a computer with a cellular automat allows to visualize and to analysize the dynamics. The key elements in the spatial dynamics are the available space A and the aging process of the developed area of the city. We think of it in terms of a transition from available and undeveloped land A to new developed areas N. With time these areas become established or old devolepments P. However development has a limited lifespan and will turn again into available land. Therefore growth is not limited to the peripherial areas. As urban sprawl continues new waves of growth appear from former developed parts within the city. According to Batty et al. this appears to be one of the keys to the solution of the problems related to suburbanisation. Finally one has to take into account that the overall growth of the city is limited.

We start with a simple mathematical model neglecting the spatiality of the problem. The total amount of available space, new developed land and old areas is given by $A(t)$, $N(t)$, and $P(t)$ respectively, with the constraint $A(t) + N(t) + P(t) = const.$ We assume that available areas are turned into new developed areas by interaction or infection with already existing new land with an infectionrate α. We can write this as

$$\frac{dA(t)}{dt} = -\alpha A(t)N(t) \tag{1}$$

In addition we assume that we have constant transitionrate γ from new developed areas to old areas. This will give us

$$\frac{dP(t)}{dt} = \gamma N(t) \qquad \text{and} \tag{2}$$

$$\frac{dN(t)}{dt} = \alpha A(t)N(t) - \gamma N(t) \tag{3}$$

A solution of these equations is shown in Fig.1. We can see the analogy to the spread of a disease if we think of $A(t)$ as the number of not infected organism. $N(t)$ refers to the number of infections where $P(t)$ is the number of organism that already passed the disease. This model is not yet what we want and especially the aging process of the city is not reflected. However it serves us in explaining the general idea and in motivating the differential equation for the spatial development in two dimensions.

$$\frac{dA(\vec{r},t)}{dt} = -\alpha A(\vec{r},t)N(\vec{r},t) + D_A\nabla^2 N(\vec{r},t) \tag{4}$$

$$\frac{dP(\vec{r},t)}{dt} = \gamma N(\vec{r},t) \tag{5}$$

$$\frac{dN(\vec{r},t)}{dt} = \alpha A(\vec{r},t)N(\vec{r},t) - \gamma N(t) \tag{6}$$

D_A is a diffusion coefficient related to the creation of available land due to the existence of new developed areas. We will use these equation as a basis for the implementation on a cellular automat. A suitable definition of a cellular automat is given by Wolfram in [2].

> "Cellular automata are mathematical idealisations of physical systems in which space and time are discrete, and physical quantities take on a finite set of discrete values. A cellular automaton consists of a regular uniform lattice (or 'array'), usually infinite in extent, with a discrete variable at each site ('cell'). The state of a cellular automaton is completely specified by the values of the variables at each site. A cellular automaton evolves in discrete time steps, with the value of the variable at one site being affected by the values of variables at sites in its 'neighbourhood' on the previous time step. The neighbourhood of a site is typically taken to be the site itself and all immediately adjacent sites. The variables at each site are updated simultaneously ('synchronously'), based on the values the variables in their neighbourhood at the preceding time step, and according to a definite set of 'local rules'."

In our case we have a quadratic grid where each site or cell can be occupied by one of the states A, N or P. We modify our existing model by introducing a fourth state V which stands for vacancy. Even within highly developed parts of the city there are for different reasons (location, shape, market) vacant areas. Each cell of the automat interacts with its eight surrounding cells as defined by some rules, which are essentially discretisation of the partial differential equation above. However we include the aging process of the established areas. After a time τ these areas will available areas again. Moreover some random element Λ_{rand} is introduced in order to break the spatial symetry. Instead of transition rates the parameters Γ, $\Phi$are now transition probabilities. Again we have a constraint: $A_{xyt}+N_{xyt}+P_{xyt}+V_{xyt}=1$, which means only one of the variables is equal to one, all others are zero. As an initial condition $N=1$ is chosen. In the variable B_{xy} the age of an etablished area is stored. The rules for the cellular automata which are iterated by every time step t are.

1. check age treshhold: if $t-B_{xyt}=\tau$ then $A_{xyt}=1$ and $P_{xy,t+1}=0$
2. determine whether available land around new developments is created:
   if $N_{xyt}=1$ and not $(N_{x\pm1,y\pm1,t}=1$ or $P_{x\pm1,y\pm1,t}=1)$
   then $A_{x\pm1,y\pm1,t+1}=1$
3. check whether available land becomes vacant:
   if $A_{xyt}=1$ or $V_{xyt}=1$ and $\Lambda_{rand}>\Gamma$ then $V_{xy,t+1}=1$
4. transfer new developed areas into established areas:
   if $N_{x\pm1,y\pm1,t}+P_{x\pm1,y\pm1,t}+V_{x\pm1,y\pm1,t}=8$
   then $P_{xyt}=1$ and $N_{xyt}=0$
5. create new developed areas out of available land:
   if $A_{xyt}=1$ and $\Lambda'_{rand}>\Phi$
   then $N_{xyt}=1$, $B_{xy}=t$, $V_{xyt}=0$ and $A_{xy,t+1}=0$

Now we have a quite simple model with only three parameters. The interesting question is, what can the model and what will happen if we play with the parameters? The M.Batty et al. who developed this model focussed their analysis on the vacancy parameter Γ and on the spread parameter Λ. The lifetime τ of development is only important on the long run. It was planned to develop a model of the growth of Ann Arbour, Michigan covering a time period of less than 20 years. Given a sufficient number of time steps however the model will show oscillotory behaviour and produce waves of new development from the core area. By sampling the parameter space $\{\Phi,\Gamma\}$, it was found that different pattern of urban sprawl could be produced as shown in Fig.2. The authors suggest to define at least four types of growth pattern: cities with low growth and low vacancy (examples might be found in Europe), cities with high growth rates and low vacancy (cities in China), cities with low growth rates and high vacancies, which might correspond to western Europe cities that have undergone Deindustrialization. High growth rates and high vacancies are characteristic for the American west. It is suggested to calibrate the model with data from different cities and to study the influence of initial conditions (growth processes which start at multiple points, influence of existing urban structure). In addition an analysis of self similarities and scaling within the urban structure is to be carried out.

References:

- [1] M. Batty, Y. Xie, Z. Sun, 'The Dynamics of Urban Sprawl'
  Centre For Advanced Spatial Analysis, Working Paper Series
  available on `http://www.casa.ucl.ac.uk`

- [2] S. Wolfram, 'The statistical mechanics of cellular automata'
  Review of modern Physics (1983) 55:601-643

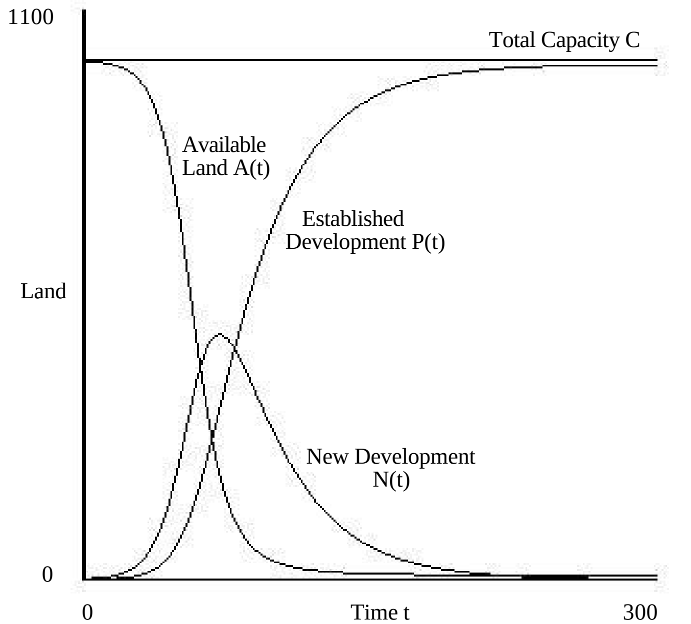
1100
Total Capacity C
Available
Land A(t)
Established
Development P(t)
Land
New Development
N(t)
0
0
Time t
300

Fig.01