Phase Slip Phenomena in Superconductors

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Abstract

The nature of a phase slip phenomenon in superconductors is described, including both phase slip center (PSC) and phase slip line (PSL) manifestation. Major models for PSC are outlined and compared with experimental evidence, with emphasis on charge imbalance waves and quasiparticle diffusion pictures of the process. Modern developments in the field are briefly outlined.

1 Introduction

The superconducting state is associated with the presence of a condensate of Cooper pairs, described by the single-valued non-zero wavefunction $\psi(\overrightarrow{r}, t)$, so that the local superconducting density of states $n_s(\overrightarrow{r}) = |\psi(\overrightarrow{r})|^2$. Conversely, whenever $|\psi| = 0$, the material is in the normal state. It is also important to note, that $\psi(\overrightarrow{r}, t)$ is analytic in these variables.

When $|\psi(\overrightarrow{r}, t)|$ is small and varies slowly in space, we can expand the free energy density in Taylor series with respect to it[1]:

$$f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\overrightarrow{A}\right)\psi\right|^2 + \frac{h^2}{8\pi}$$

where f_n is the free energy of the normal state. Writing $\psi = |\psi|e^{i\phi}$ allows us to rewrite this as:

$$f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left(\hbar^2(\nabla|\psi|)^2 + \left(\hbar\nabla\phi - \frac{e^*}{c}\overrightarrow{A}\right)^2|\psi|^2\right) + \frac{h^2}{8\pi}$$

For $|\psi|$ that varies slowly in space and in London gauge we can reduce this to:

$$f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{m^* v_s^2}{2}|\psi|^2 + \frac{\overrightarrow{h}^2}{8\pi}$$

where the supercurrent velocity is:

$$\overrightarrow{v_s} = \frac{1}{m^*}\left(\hbar\nabla\phi - \frac{e^*\overrightarrow{A}}{c}\right)$$

Following F.London, let us also define the fluxoid, for each normal region inside a superconductor, as:

$$\Phi' = \underbrace{\oint \overrightarrow{A}\cdot d\overrightarrow{s}}_{Flux\,\Phi = \oint \overrightarrow{h}\cdot d^2\overrightarrow{r}} + \frac{m^*c}{e^*}\oint \overrightarrow{v_s}\cdot d\overrightarrow{s} = \frac{c}{e*}\oint\left(m^*\overrightarrow{v_s} + \frac{e^*\overrightarrow{A}}{c}\right)\cdot d\overrightarrow{s} = \frac{\hbar c}{e*}\oint \nabla\phi\cdot d\overrightarrow{s}$$

where the integrals are along a simple closed path that encircles the normal region.

Since ψ is single-valued, the value of the phase must be same modulo 2π at the ends of any closed loop, which means that:

$$\oint \nabla\phi\cdot d\overrightarrow{s} = 2\pi n \;\Rightarrow\; \Phi' = \frac{hc}{e^*}n = \Phi_0 n$$

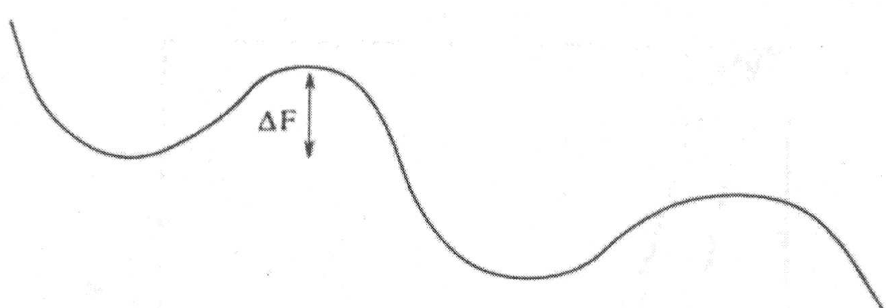
where n is an integer, and $\Phi_0 = \frac{hc}{e^*} = \frac{hc}{2e} = 2.07\times 10^{-7}G - cm^2$.

If the loop has cylindrical symmetry, so that both $\overrightarrow{h}$ and $\overrightarrow{v_s}$ are constant around the loop, with a hole of radius R and the superconducting wall sufficiently thin that we may ignore the difference between the fields in the hole and inside the wall, then we have from definition of Φ':

$$\overrightarrow{v_s} = \frac{e^*}{m^*c2\pi R}\left(\Phi_0 n - \pi R^2\overrightarrow{h}\right) = \frac{h}{m^*2\pi R}\left(n - \frac{\Phi}{\Phi_0}\right)$$

The thin wall condition is $thickness\; d < \lambda_L$.

Therefore the free energy has minima for each value of n, with the global minimum associated with the value of n, such that $\left|n - \frac{\Phi}{\Phi_0}\right| \leq \frac{1}{2}$, as is illustrated in Fig.1[2]:



Schematic free energy diagram

Fig. 1

2 Phase slip center[12]

As noted by Little[3], for a system to go from a state indexed by n_1 to another state indexed by n_2 it must pick up $n_2 - n_1$ windings of the phase. For example, if we in some way created a supercurrent through the loop, and then removed external driving, the global minimum for the system would be associated with $n = 0$, even though we have excited the system to $n \neq 0$. We therefore expect the system to relax to its global minimum by unwinding its phase. However, phase cannot change discontinuously, since ψ is analytic in time, so the only way for the phase to unwind is for $|\psi|$ to vanish at least at one point along the loop. More precisely superconductivity is destroyed in the normal region of size $\sim \xi$, so that the energy necessary to drive this region into normal state in the low current limit is $\frac{8\sqrt{2}}{3}\frac{{H_c}^2}{2\mu_0}\xi\sigma$, where σ is the cross-sectional area of the loop[4]. Such a localized phase transition is referred to as a phase slip center.

A very rough picture of a phase-slip center may be obtained if we look at the particle approach. In this picture, there is a place in a superconductor which has a lower T_c due to, for example, impurities. As we increase the supercurrent, it will first exceed the critical current density in such a "weak link" region. Then Cooper pairs are broken up and the supercurrent is carried by normal electrons. Such electrons will loose their momentum through inelastic collisions and therefore dissipate excess energy we supplied. Once that happens, superconductivity can be reestablished. The advantage of this picture is that it elucidates the essence of a phase-slip event, namely the transition between quasiparticle states and condensate, in a region that is at least locally out of equilibrium.

Rieger, Scalapino and Mercereau [5] pointed out the fundamental importance of the difference between the chemical potentials of quasiparticles and the condensate. They specify Cooper pair and quasiparticle chemical potentials respectively as follows:

$$\mu_p = \frac{i\hbar}{4|\psi|^2}\left(\psi^*\frac{\partial\psi}{\partial t} - c.c.\right)$$
$$\mu_p - \mu = \frac{m\xi^2}{4|e|\tau}\frac{\nabla\cdot\overrightarrow{j_s}}{|\psi\}^2}$$

where τ is characteristic relaxation time. The divergence of $\overrightarrow{j}_s$ implies that pairs are forming or breaking up, which requires a difference of chemical potentials between the condensate and the quasiparticles.

RSM also proposed a model for PSC, in which normal and supercurrents coexist near the "weak link", so that the normal current is given by $\overrightarrow{j} - \overrightarrow{j_s}$. They also assumed the effective resistance of the "weak link" to be the residual resistance of the normal phase. The order parameter in their model declines to zero prior to phase-slip event and then grows back, so that the $\overrightarrow{j_s}$ does not decay toward zero when the "weak link" becomes a PSC, but rather it oscillates. Indeed, they derive an analytical approximation for the supercurrent as a function of phase difference at the ends of a superconducting path:

$$j_s = \frac{j_c}{2}(1 + cos(\triangle\phi))$$

3 The Skocpol, Beasley, Tinkham (SBT) model[6]

This theory is for quasi-one-dimensional homogeneous superconductors, e.g. whiskers of Sn. Then the normal and supercurrent densities are given by

$$j_n \approx \frac{1}{e\rho_n}\frac{d\mu}{dx}$$

$$j_s = \frac{2e\hbar}{m^*}|\psi|^2\frac{d\phi}{dx}$$

SBT explicitly neglect deviations from electrical neutrality, so they have $j_n + j_s = j$. They assume that variations of the order parameter occur on the length scale of $\sim 2\Lambda$, where Λ is the quasiparticle diffusion length, essentially a quasiparticle mean free path. The order parameter is assumed to be identically zero only at the center of the PSC. SBT assume that all of the applied voltage gets dropped right at the core of PSC, so that in their steady state approximation, there is no variation in the time-averaged value $<\mu_p>$ inside superconducting regions. However $<\mu_p>$ experiences sharp steps at each PSC. Of course, the actual μ_p will vary, corresponding to the variations of the superconducting phase, as given by the Josephson relation $\mu_p = -\frac{\hbar}{2}\frac{\partial\phi}{\partial t}$.

Since the voltage is only dropped at the core of the PSC, it is a reasonable approximation that:

$$eV = \triangle\mu$$

$$i.e.\ V = \frac{\triangle\mu}{e} \approx 2\rho\Lambda(j - j_s) = \frac{2\rho\Lambda}{\sigma}(I - I_s)$$

Notice that since the voltage drop is due to normal current only, the above relation uses μ rather than μ_p.

Based on the expected shape of μ_p during the first half of the cycle (i.e. when the supercurrent gets accelerated), SBT modeled it as:

$$\mu_p = \frac{\triangle\mu_p}{2}\left(1 - tanh\left(\frac{x - x_{psc}}{\pi\xi}\right)\right) + <\mu_p>_{x>x_{psc}}$$

The μ_p for the deceleration part is assumed to be close to asymptotic values away from the PSC, but with a gradient which is the $-\gamma$ times that of the above expression and is discontinuous at $x = x_{psc}$. Here the parameter γ specifies the relative times for the acceleration and deceleration parts of the cycle. SBT used $\gamma \gg 1$, which corresponds to an instantaneous deceleration, a "snap back" of the phase.

SBT further use the Pippard, Shepherd and Tindall result[6] : $<\mu> - <\mu_p> = \Lambda^2\frac{d^2}{dx^2}(<\mu> - <\mu_p>)$. They solve this equation on both sides of the PSC, such that on both sides they assume the presence of bulk regions (say at

x_l and x_r), where $<\mu>=<\mu_p>$. They then have, say on the left side of the PSC:

$$<\mu> - <\mu_p> = Usinh\left(\frac{x - x_l}{\Lambda}\right)$$

So now, assuming $x_l \gg \Lambda$ and $x_r \gg \Lambda$:

$$V = \frac{\triangle\mu}{e} = \rho\Lambda\left(tanh\left(\frac{x_{psc} - x_r}{\Lambda}\right) + tanh\left(\frac{x_l - x_{psc}}{\Lambda}\right)\right)(j - <j_s>) \approx 2\rho\Lambda(j - <j_s>)$$

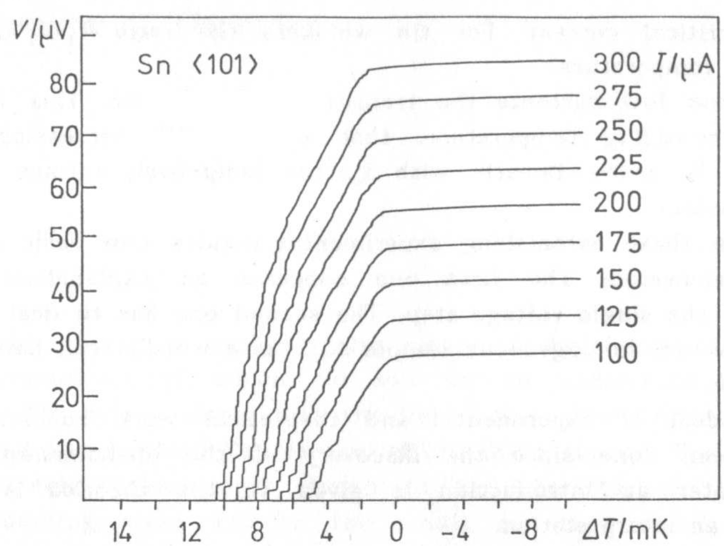
Then:

$$\frac{dV}{dj} \approx 2\rho\Lambda$$

This then suggests that each PSC contributes a differential resistance of $\frac{2\rho\Lambda}{\sigma}$.

Since each phase slip center can carry about $0.6I_c$, it follows that the experiments should show this excess current. Furthermore, as more and more PSCs develop, the dissipative processes should contribute to larger and larger differential resistance, so that the I vs. V curve is predicted to have step-like nature, with each step being a straight line with a $\sim 0.6I_c$ zero voltage intercept.

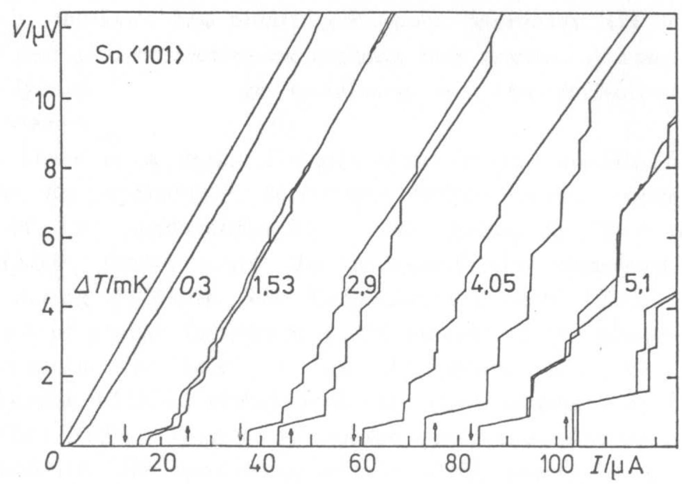
This is indeed observed experimentally, as illustrated in Fig. 2a and 2b



V−T transition curves of a Sn whisker at different fixed currents I

Data measured by J.D.Meyer (1973)
$\triangle T = T_c - T$
Sample length $L = 8 \cdot 10^{-2}$
Sample cross-section area $A = 1.93\ \mu m^2$

Fig. 2a



V–I hysteretic curves for several fixed temperatures $T=T_c-\triangle T$

Data measured by J.D.Meyer (1973)
$\triangle T=T_c-T$
Sample length $L=8\cdot 10^{-2}$
Sample cross–section area $A=1.93\ \mu m^2$

Fig. 2b

Several groups also conducted experiments where $<\mu>$ and $<\mu_p>$ were directly measured. Dolan and Jackel[7] measured both $<\mu>$ and $<\mu_p>$in long microbridges of tin and indium, using opposed sets of normal and superconducting probes with oxide barrier tunnel junctions, and showed that indeed $<\mu>$ varies smoothly $<\mu_p>$ varies abruptly at the PSC. Other similar experiments by Aponte and Tinkham[8] and Stuivinga et.al.[9] all confirm the predictions of the SBT model.

4 The Kadin, Smith Skocpol (KSS) model[10]

The KSS model is more general and detailed than SBT model, taking into account AC effects and the dynamics at the core of a PSC. Of fundamental importance to KSS model is the quantity of charge imbalance:

$$Q^* = 2eN_0(\mu_p - \mu_f)$$

where μ_f is the chemical potential of electrons in the normal state, and N_0 is the local density of states for one spin direction at Fermi energy. KSS showed that this quantity satisfies the charge imbalance wave equation:

$$\frac{d^2}{dx^2}Q^* = \frac{\tau_o\tau_e}{D\tau_{Q^*}}\frac{\partial^2}{\partial t^2}Q^* + \frac{\tau_o+\tau_e}{D\tau_{Q^*}}\frac{\partial}{\partial t}Q^* + \frac{1}{D\tau_{Q^*}}Q^*$$

This is the well-known telegraph equation for transmission lines:

$$\frac{\partial^2}{\partial x^2}E = LC\frac{\partial^2}{\partial t^2}E + (RC+GL)\frac{\partial}{\partial t}E + RGE$$

with:
$R = \frac{\rho_n}{\sigma}$ - normal resistance per unit length
$L = \frac{\tau_0 \rho_n}{\sigma}$ - $R\tau_0$, where τ_0 is the supercurrent response time
$C = \frac{A\tau_e}{\Lambda^2 \rho_n}$- $G\tau_e$, where τ_e is inelastic electron-phonon collision time
$G = \frac{A}{\Lambda^2 \rho_n}$ - kinetic inductance of supercurrent.

Note also that τ_{Q^*} is is the time scale characterizing relaxation by changes in the distribution function alone. Near T_c:

$$\tau_{Q^*} = \frac{4k_B T}{\pi \Delta}\tau_e$$

Finally D is the normal electronic diffusion constant.

Now the essential idea for the modeling of the PSC is to use an equivalent transmission line circuit with an ideal Josephson element inserted in the inductive line. the Josephson oscillator is assumed to be of zero length and resistance and it is assumed to have a current-phase relationship of the form:

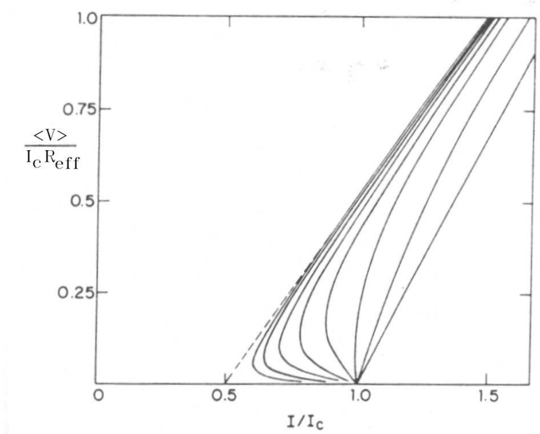
$$I_s(x=0,t) = I_c f(\triangle\theta(t))$$

For high DC voltages applied to the sample, $I \gg I_c$, so that I_n does not need to compensate for I_s and therefore th voltage across the Josephson element is nearly constant. Therefore charge imbalance waves will only be a small disturbance of the DC behavior, so that SBT picture is reproduced. KSS model even predicts that $<\mu_p>$ is constant inside the superconducting regions and that $<\mu>$ and I_n decay exponentially within the length scale Λ.

In a more general case, KSS model gives the following I-V characteristics:

$$I = \begin{cases} I_c + \left(1-\sqrt{\frac{\tau_e}{\tau_0}}\right)\frac{<V>}{R_{eff}} & <V> \ll I_c R_{eff} \\ <f> I_c + \frac{<V>}{R_{eff}} & <V> \gg I_c R_{eff} \end{cases}$$

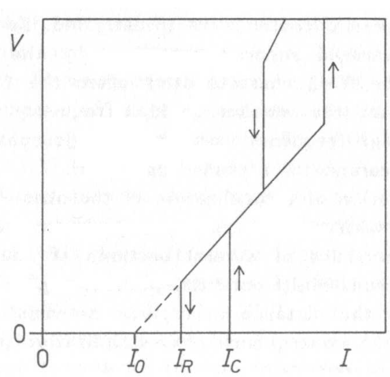
where $f(\triangle\phi)$ is the current phase relation, and $R_{eff} = \frac{V_c}{I_c}$ is the effective resistance.



I-<V> characteristics in the high frequency limit, for the linear current-phase relation. The curves are scaled so that they all have the same critical current and high voltage differential resistance. τ_e/τ_0 are respectively from right to left: 1024, 256, 64, 4, 1, 1/4, and 1/16.

Fig. 3

This important KSS model result predicts hysteresis in the normal-superconducting transition for many I-V branches (see Fig. 3), namely when $\tau_e > \tau_0$. We see that the normal state will be established at I_c as we raise the current, but it will go back to superconducting state at a current associated with the point where th curves begin to bend back toward I_c. Indeed, such hysteretic behavior is observed and is moreover one of the characteristic features of the transition, e.g. Fig. 4



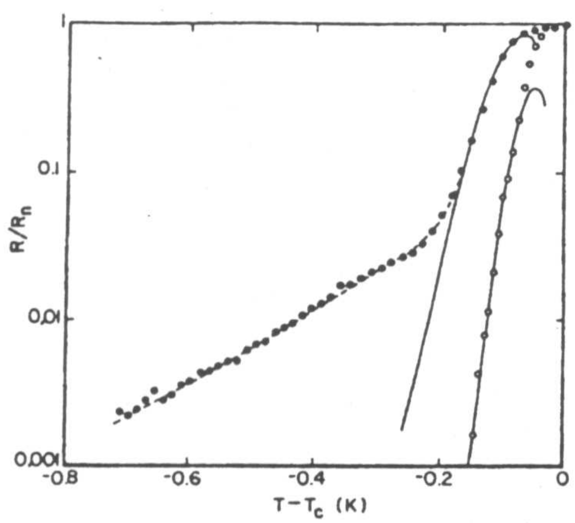
Sketch of the first (and part of the second) voltage step in the I-V characteristic of the quasi-1D superconductor

Fig. 4

5 Modern Developments

In recent years, PSC research proceeded in several directions. There is interest in understanding phase-slip phenomena in non-superconducting materials where a condensate is present, such as charge density wave materials. Another direction of research is to make samples smaller, truly one-dimensional. Most experimental studies before 1990s were done using samples which satisfied the condition $d < \lambda_L$, but not the more strict condition $d < \xi$. Consequently, researchers were limited to a very tight range of temperatures (several mK) near T_c. N.Giordano[2] has reported studies of truly small systems satisfying the condition for one-dimensional samples. He observed a resistive state that goes far below T_c (˜.5K).

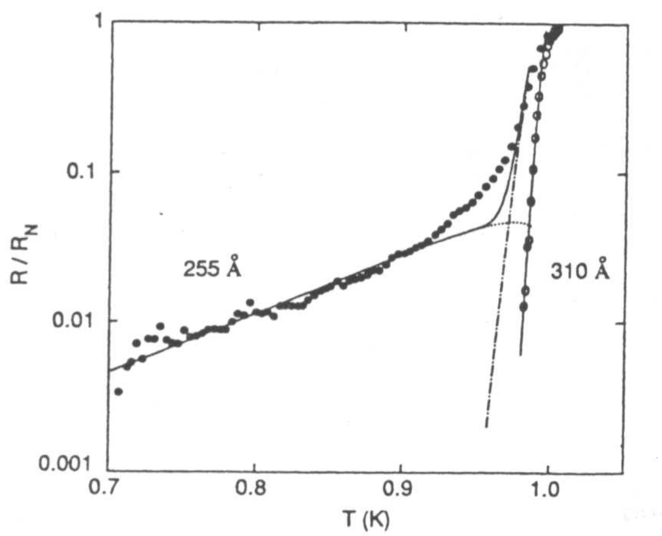
He notes that the resistance data only fits thermal activation models close to T_c, and displays fundamentally different behavior at lower temperatures, as can be seen from Fig. 5a and 5b:



Resistance as a function of temperature for 410 Å (closed symbols) and 745 Å (open symbols) In wires. The solid curves are the prediction of thermal activation theory, while the dashed curve is a fit which includes the contribution of quantum tunneling.

Data measured by N.Giordano

Fig 5a



Resistance as a function of temperature for 255 Å (closed symbols) and 310 Å (open symbols) PbIn wires. The solid curve for the 310 Å sample and the dot–dashed curve for the 255 Å sample are the predictions of thermal activation theory. The solid curve for the 255 Å sample includes the contribution of quantum tunneling.

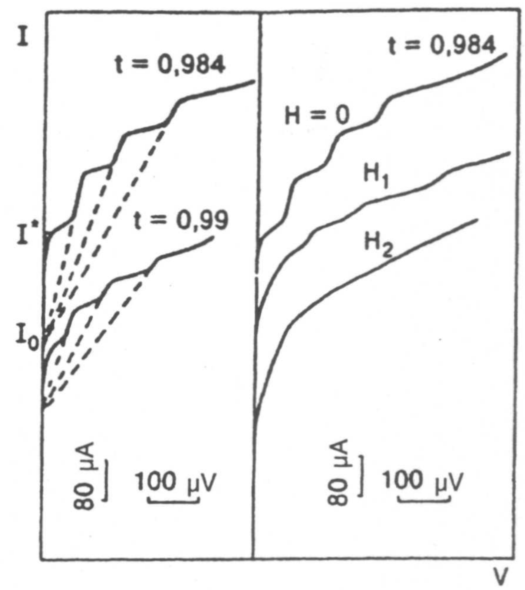
Data measured by N. Giordano

Fig. 5b

The proposed tentative explanation for this phenomenon is a phase slip due to quantum tunneling through the free energy barrier. The postulate of this theory is that the system is underdamped, i.e. there is little interaction with the environment. The suggested behavior of the resistance is then $R \sim \exp\{\frac{-A\triangle F}{\hbar w}\}$, where $\triangle F$ is the energy barrier and w is the frequency of underdamped oscillations.

5.1 Phase slip lines[11]

Phase slip lines are direct analogs of PSC, in the case of quasi-two-dimensional materials (i.e. $d \gg \lambda_L$). The I-V curves for such films also show a step structure with a well defined excess current.



Step structure of the intial region of I−V characteristics for a Sn film with w>λ_L at two tempatures (a) and in various magnetic fields (b) H_2>H_1 ; Sivakov (1986)

This is observed in zero external magnetic field ($h \sim 10^{-3}Oe$). Experiments similar to those of Dolan and Jackel were performed and it was established that μ and μ_p differ along a PSL. A detailed theory of the process is not available due to the difficulty of solving 2D time-dependent GL equation in the presence of both PSL and vortices as dissipation mechanisms.

6 Summary

The non-equilibrium phenomena in superconductors still present challenges for fundamental research. Probing such phenomena allows one to measure important physical constants associated with superconductivity, e.g. relaxation times for various processes, as well as to advance our understanding of the superconducting state. Last but not least, phase slip centers and phase slip lines show an array of effects which may be of engineering interest as well.

7 References

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