Can a Solid be a Superfluid ?

Term Paper for Physics 463

Spring 1999

Vassiliy Lubchenko

Abstract

It has been proposed that in certain solids at very low temperatures, there might be delocalized excitations forming a Bose condensate. As a result, the density of the solid could have a finite superliquid component. This phase represents another state of matter called "supersolid". In spite of a large amount of theoretical and computational work suggesting a possibility of such a phase, there is no unambiguous experimental proof that the supersolid exists. In this paper, I will overview some theoretical motivations for the search of the superfluid component in a solid as well as some of the more optimistic experimental results.

1 Introduction

In this paper, I will give highlights on the current state of the human effort to detect another state of matter - the so called "supersolid" state. It is beleived [1, 12] that there are conditions under which a certain (very small) fraction of matter in a crystalline solid can form a superfluid. This situation in which long-range crystalline and superfluid order coexist is called the "supersolid" phase. In this paper I will attempt to give you some essential ideas suggesting the existence of such an exotic phase as well as current experimental progress in the field.

The paper is organized as follows: in the next section I will state general considerations on the issue following the review of Misel [1], which I found the best starting point for acquaintance with the subject. Subsequently, I will outline original propositions of the supersolid phase according to [5],[4],[3] (given here not in the chronological order, but rather on the basis of coming from very general grounds to more specific models). Next, in Section 3, I will highlight the results of Lengua and Goodkind [7] on sound attenuation in solid ^{4}He with their interpretation of the observed resonant behaviour as to infer the existence of a coherent ground state of the gas of mobile defects. Even if there was no superfluidity in their experiment, their results undoubtedly show the existence of otherwise non-identified excitations in the hcp ^{4}He. Lastly, I will pose some questions that I found unanswered in the literature.

2 Why superfluid solid?

The coexistence of a solid and a superliquid phase in one volume does not sound very conventional, but think of this: what if a certain fraction of molecules in a crystal is mobile enough to communicate with each other so as to form a gas on the background of the lattice? Then, provided the molecules obey the Bose-Einstein statistics, they will form a condensate at a low enough temperature. In reality, atoms are mutually impenetrable and by the very nature of the solid phase cannot move freely through the lattice; in fact, as shown in [2], BEC (Bose-Einstein Condensation) cannot occur in a perfect crystal. (I assumed that diffusion cannot give rise to a collective excitation by the very incoherent nature of diffusion.) On the other hand, if there is a sufficient density of vacancies in the lattice (or other defects such as dislocations), they could move in the bulk without having to physically interchange the molecules and form a gas of quasi-particles with presumably Bose statistics and all the consequences mentioned above. I will return to this again, but it is clear even now that to have mobile molecules at very low temperatures needed for BEC or to have enough residual volume to allow for the vacancies we need a solid consisting of very light molecules, so that they have very large zero-point vibrations (the thermal vibrations are essentially frozen out at these temperatures). What is a good candidate [1]? A convenient criterion for a solid to be quantum comes from de Boer [9]. One thinks that the properties of the solid are largely determined by quantum laws if the amplitude of zero point motion is comparable to the lattice spacing. If molecules interact through a Lennard-Jones potential $U(r) = 4\,\epsilon\,[(\sigma/r)^{12} - (\sigma/r)^{6}]$, then the de Broglie wave-length of their relative motion (with an energy $\sim \epsilon$) scales as $h/\sqrt{m\epsilon}$. Lattice spacing is σ, so that the degree of quantumness of the solid is expressed by the so called de Boer parameter $\Lambda \equiv h/\sigma\sqrt{m\epsilon}$. ^{3}He has the biggest Λ of $\simeq 3.1$, followed by ^{4}He with $\Lambda \simeq 2.7$, H_2 ($\Lambda \simeq 1.7$) etc. ^{3}He undergoes a nuclear magnetic ordering at $\sim 1mK$, so we are left with ^{4}He crystal as the most likely system to observe a supersolid state. Another part of this argument is that one is more likely to find the vacancies closer to the melting line of the crystal. Current experiments indicate that the concentration of the defects in solid ^{4}He does not exceed 10^{-6}.

How might a superfluidity in crystals come about?

2.1

Let us first consider a general argument on how a finite superliquid fraction of a solid can exhibit itself in an experiment according to Leggett [5]. Suppose, we make a cylindrical annulus (hollow cylinder) out of ^{4}He with a radius R and thickness $d \ll R$ and rotate it at constant angular velocity ω. Then the free energy in the rest frame is

$$F(\omega) = F_0 + \frac{1}{2} I_0 \omega^2 + \Delta F(\omega), \tag{1}$$

where $F_0 \equiv F(0)$ is the free energy in the abscence of rotation and $I_0 = NmR^2$ is the classical moment of inertia. $\Delta F(\omega)$ is associated with the superfluid component in the density and assumed to be even and periodic with period ω_0:

$$\omega_0 = \frac{\hbar}{\alpha m R^2}. \tag{2}$$

The periodicity has to do with the quantization of angular momentum in the superliquid (also cf. Little-Parks experiment, where free energy is a periodic function of the magnetic flux). Importantly, α is of the order unity. For $\omega < \omega_0/2$, the function $\Delta F(\omega)$ is of the form:

$$\Delta F(\omega) = -\frac{1}{2}(\rho_s/\rho) I_0 \omega^2, \tag{3}$$

which defines the superliquid fraction (ρ_s/ρ). Putting together (1) and (3) we see that if there is a superliquid component in the solid, the system will lower its free energy by maintaining some part of the substance at rest, which will show up macroscopically as a decrease in the moment of inertia.

The argument for estimation of the superliquid fraction (ρ_s/ρ) goes as follows [5]. Let θ_i' be the angular cordinates of the He atoms in the obviously chosen cylindrical coordinates, then the wave function has to satisfy a periodicity condition to be single-valued:

$$\Psi_0(\theta_1, \theta_2, \dots, \theta_i + 2\pi, \dots, \theta_N) = \Psi_0(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_N). \tag{4}$$

If the annulus is rotated with a constant angular velocity ω, the angles θ_i in the rest frame are expressed through those in the rotating frame as

$$\theta_i = \theta_i' + \omega t, \tag{5}$$

and the wave function is

$$\Psi(\theta_i) = \Psi'(\theta_i') \exp\left(im\omega R^2 \sum_i \theta'(t)/\hbar\right) \tag{6}$$

(cf. [10] §17). Therefore the periodicity condition (4) is modified to:

$$\Psi(\theta_1, \theta_2, \dots, \theta_i + 2\pi, \dots, \theta_N) = \exp\left(-2\pi i m\omega R^2/\hbar\right) \Psi(\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_N). \tag{7}$$

We continue the discussion for the case when $T = 0$. Assume that the following Hamiltonian describes the system:

$$\hat{H} = -\frac{\hbar^2}{2m} \sum_i \nabla_i^2 + \frac{1}{2} \sum_{ij} U(\mathbf{r}_i - \mathbf{r}_j) + \sum_i V(\mathbf{r}_i). \tag{8}$$

Energy of the system is found by minimizing the expectation of the Hamiltonian subject to the periodicity condition for the wave function. It follows then that

$$\Delta F(\omega) = -[E_{min}(w) - E_0], \tag{9}$$

where E_0 is found by minimizing energy expectation value subject to B.C. at rest (4) and E_{min} corresponds to the rotating ones from (7). From (9) and (3) one finds:

$$\rho_s/\rho = \lim_{\omega \to 0} I_0^{-1} \partial^2 E_{min}(\omega)/\partial\omega^2. \tag{10}$$

If therefore $E_{min}(\omega)$ is independent of ω, the system cannot be superfluid.

Also, it follows from (7) that $E_{min}(\omega) = E_{min}(\omega + \hbar/mR^2)$. This argument forces α in (2) to be integer. Then (2) tells us that the system will not become superfluid, if $|E_{min}(w) - E_0| < \hbar^2/mR^2$.

The system cannot remain at the energy E_0 and rotate at the same time because the wave function Ψ_0 corresponding to E_0 does not obey the new boundary conditions, *unless* it vanishes at least at one point on any path inside the material so that it can switch the phase to match the modified B.C. (the path has to make a full circle around the axis, of course). If all atoms in the bulk are localized, the latter condition of disconnectedness of Ψ_0 is easily fullfilled. However, for a crystal with large quantum fluctuations, it is hard to imagine that there is not a *single* path where the function is non-zero at each point. The argument above is of a very general nature and uses only symmetry principles.

2.2

Chester [4] has made the following argument on the possibility of BEC in the zero-momentum state that exhibits crystalline order (BEC is not sufficient for superfluidity but seems to be necessary). Consider a wave-function of the following kind for a Bose gas:

$$\Psi_N = \prod_{i\neq j} f(r_{ij}) = \exp\left[-\frac{1}{2}\sum_{i\neq j} u(r_{ij})\right] \tag{11}$$

where

$$\begin{aligned} u(r) &= \infty,\ r < a \\ u(r) &> 0,\ a < r < b \\ |u(r)| &< |Ar^{-3-\epsilon}|,\ b < r, \end{aligned} \tag{12}$$

with A, a, b and ϵ positive constants. $u(r)$ has a hard core, bounded below and has a finite range.

One can show [4] that the wave-function (11) has a Bose-Einstein condensate in the zero-momentum state.

What Chester suggested was that Ψ_N leads to a probability distribution

$$P_N = \Psi_N^2 = \exp\left[-\sum_{i\neq j} u(r_{ij})\right], \tag{13}$$

which is nothing but a classical probability density for the motion of the particles in an effective mutual potential $\phi(r) = k_B T\, u(r)$. If we make a basic assumption that the Gibbs distribution is capable of describing all the phases of the same system, it naturally follows that potential $\phi(r)$ will provide a crystalline order at high enough densities, which has been seen in simulations.

As a result, we conclude that BEC can occur in a crystal. To add on my part, it seems almost intuitive that if there is a condensate uniform in space throughout the whole sample (here, $k_{BEC} = 0$), we will have a superfluid, albeit it is not at all clear what the critical velocity will be (cf. $2D$ case).

As in the previous argument, importance of large density fluctuations in the system is hard to overestimate.

2.3

Historically, Andreev and Lifshitz were the first to comment on the possibility of the condensation of a gas of "defectons", or "impuritons", as they named the quasiparticles associated with the motion of the vacancies present in the lattice [3]. Here is some of their reasoning.

If we have a number of defects in the lattice, classically, they will be localized with a certain energy $E_0 > 0$. Quantum mechanically, a chance to escape (tunnel away) gives rise to the appearance of a band with a finite width on the order of the inverse time spent by a defect on a particular site (band is centered at E_0). In a very quantum crystal (de Boer's $\Lambda \sim 1$), defects are very mobile so that the bandwidth $\Delta\epsilon$ could get large enough to move the ground state $\epsilon_0 = E_0 - \Delta\epsilon/2$ (bottom of the band) low enough to make some kind of reorganization of the crystal energetically favorable. What kind of reorganization?

Let the defecton dispersion relation be

$$\epsilon(k) = \epsilon_0 + k^2/2M, \tag{14}$$

where M is the effective mass, which we estimate by noting that the band width is the energy difference between the edge and the middle of the Brillouin zone whose width in its turn is on the order of a^{-1} the inverse lattice spacing, that is $M \sim (\Delta\epsilon a^2/\hbar^2)^{-1}$.

By its meaning, ϵ_0 is the difference between energies on different lattice sites. At a certain pressure with a given volume, or, conversely, at a certain volume V_0 with pressure given, ϵ_0 will tend to zero. One expands ϵ_0 in powers of $(V_0 - V)$:

$$\epsilon(k) = \lambda\frac{V_0 - V}{V} + k^2/2M, \tag{15}$$

where λ is positive for definiteness. If $V < V_0$ then the number of defectons is proportional to $\exp(-\epsilon_0/k_B T)$ and tends to zero as $T \to 0$. On the other hand, as $V > V_0$, the defectons will condense into $\mathbf{k} = 0$ state. If one assumes that the defecton gas is rare (Van der Waals), he can use the the following expression for its energy (cf. [11] section 78):

$$E = \lambda\frac{V_0 - V}{V}N + \frac{2\pi a}{M}\frac{N^2}{V}, \tag{16}$$

where N is the number of defectons and a is the amplitude for their mutual scattering. We assume that it is energetically hard to create two defectons on one site, as it will involve creation of too much empty volume. This corresponds to repulsion between defectons, i.e. $a > 0$. We find the equilibrium number of defectons N_{eq} by optimizing energy with respect to N and obtain:

$$\frac{N_{eq}}{V} = -\frac{M\lambda}{4\pi a}\frac{V_0 - V}{V_0}. \tag{17}$$

Subsequently,

$$E = -V\frac{M\lambda^2}{8\pi a}\left(\frac{V - V_0}{V_0}\right)^2. \tag{18}$$

We see that a continuous (second order) transition occurs at $V = V_0$. The order parameter is the square root of the number of defectons or, alternatively, the residual volume due to the emergence of defectons as follows from (17). We see that it might be energetically favorable for a quantum crystal to acquire a macroscopically large difference between the number of lattice sites and the number of atoms, which results in emergence of a gas of quasi-particles-defectons which will Bose-condense giving rise to a possibility of superfluidity in the system.

I defer further discussion till the concluding section and legitimately ask:

3 Experimental Evidence ?

3.1

There is no positive experimental evidence for the existence of a superfluid state so far. Attempted approaches include among others[1]:

- plastic flow in crystals in which physical objects were moved
- mass flow through a weak link which was subjected to a chemical potential difference between two mass reservoirs (technique reminiscent of one used for liquid He II)
- use of a sensitive torsional oscillator to search for a change in the moment of inertia of the system (cf. subsection 2.1)
- thermodynamic measurements of pressure as a function of temperature.

Table 1 summarizes the experimental conditions and obtained limits on the superfluid density ρ_s/ρ and critical velocity v_c (see end of paper).

Possible reasons for the abscence of good news are that either the experiments were performed at a temperature higher than the critical temperature T_c, or the velocity of the attempted mass transport was higher than the critical one v_c. Thermodynamic measurements mentioned above do not depend on the latter constraint, but the interpretation of the results depends on whether the postulated gas of the vacancies is weakly or strongly interacting. For an ideal Bose gas one should recover the following contributions to $(\partial P/\partial T)_V$ from the vacancies with concentration n_v:

$$\begin{aligned} \left(\frac{\partial P}{\partial T}\right)_V^{vac} &= 1.00 n_v R \ \text{ for } T \gg T_c \\ \left(\frac{\partial P}{\partial T}\right)_V^{vac} &= .51 n_v R \ \text{ for } T = T_c \\ \left(\frac{\partial P}{\partial T}\right)_V^{vac} &= .51 n_v R (T/T_c)^{2/3} \ \text{ for } T < T_c, \end{aligned} \tag{19}$$

with

$$T_c = \left(\frac{h^2}{2\pi m k_B}\right)\left(\frac{n_v}{2.612}\right)^{2/3}. \tag{20}$$

T_c might be pretty small, but at least, with improved resolution one might be able to extract the value of n_v from the pressure measurements after substraction of the phonon contribution:

$$\left(\frac{\partial P}{\partial T}\right)_V^{ph} = \frac{234\gamma R}{V}\left(\frac{T}{\Theta_D}\right)^3, \tag{21}$$

where Θ_D is the Debye temperature ans γ is the Grünisen constant. So far, no anomalies in these measurements have been observed.

3.2

However, some promising results have been obtained by Lengua and Goodkind [7]. They measured ultrasound attenuation α and velocity shift $\Delta v/v_0$ in hcp ^{4}He as functions of temperature. The important part of their results is presented in figures 1 and 2.

There are also non-resonant contributions to α and $\Delta v/v_0$ due to interactions of the ultra-sound with thermal phonons and static lattice defects. Those agree very well with other measurements and were substracted to emphasize the remaining resonant features. The authors argue that the observed peaks are due to the interactions of the probe sound waves with the superliquid component, which they argue is a Bose-condensed gas of the defectons (quasi-particles associated with propagating defects).

In what follows I will try to give a flavor of their phenomenological approach, which I find instructive. They postulate that the sound wave and the new collective mode propagate and interact according to the so called traveling wave formalism:

$$\begin{aligned}\nabla^2\xi &= \frac{1}{v_0^2}\ddot{\xi} + \eta\dot{\xi} + \zeta\ddot{\Xi} + \mu\dot{\Xi} \\ \nabla^2\Xi &= \frac{1}{c^2}\ddot{\Xi} + \beta\dot{\Xi} - \epsilon\ddot{\xi} - \nu\dot{\xi}\end{aligned} \tag{22}$$

with v_0 and c being speeds of sound and the superliquid component respectively. Eqns. (22) simply represent the most general kind of linear coupling between two waves. Small values of μ and ν happened to work well to explain the observations, so I will assume they are equal to zero. I cite the resulting formula for the change in the sound attenuation due to wave interaction valid at low coupling between the waves:

$$\Delta\alpha = \frac{1}{2}v_0\frac{\epsilon\xi(\eta-\beta)}{\left(\frac{1}{v_0^2}-\frac{1}{c^2}\right)^2 + \frac{1}{\omega^2}(\eta-\beta)^2} \tag{23}$$

Eq. (23) shows that to explain the peaks in figs.1 and 2 we have to look for resonances of the kind

$$c(T)|_{T=T_R} = v_0, \tag{24}$$

where T_R's are temperatures at the centers of the resonance peaks. c depends on the temperature through the temperature dependence of the defect concentration. To estimate the latter we adopt the model of defectons postulated by Andreev and Lifshitz. Spectrum of the defectons (identical to (14) with some change of notation):

$$\hbar\omega(q) = k_B\Phi + \hbar^2 q^2/2M. \tag{25}$$

At a finite temperature T, we look for the number of thermally activated defectons by using Bose-Einstein statistics:

$$N = V\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{e^{\hbar\omega/k_BT}-1}. \tag{26}$$

This gives for their concentration:

$$n(T) = \left(\frac{Mk_B}{2\pi\hbar^2}\right)^{3/2} T^{3/2}F_{3/2}\left(\frac{\Phi}{T}\right), \tag{27}$$

where

$$F_\sigma(z) \equiv \frac{1}{\Gamma(\sigma)}\int_0^\infty \frac{y^{\sigma-1}dy}{e^{y+z}-1}. \tag{28}$$

We now treat the defecton propagation analogously to elastic waves in the way that we introduce a defecton displacement field $\mathbf{s}$, then excess defecton density at a particular location is $\Delta n = -n\nabla\mathbf{s}$ and excess energy due to it is

$$\Delta V = \int V(\mathbf{r}-\mathbf{r}')dn(\mathbf{r}') = -n\int V(\mathbf{r}-\mathbf{r}')\nabla'\cdot\mathbf{s}(\mathbf{r}')d^3\mathbf{r}'. \tag{29}$$

The resulting equation of motion for a unit volume reads then:

$$M\frac{\partial^2\mathbf{s}}{\partial t^2} = \nabla(\Delta V) = n\int \nabla V(\mathbf{r}-\mathbf{r}')\nabla'\cdot\mathbf{s}(\mathbf{r}')d^3\mathbf{r}'. \tag{30}$$

Plugging in a plane wave solution

$$\mathbf{s} \propto \hat{\mathbf{k}}e^{i(\mathbf{kr}-\omega t)} \tag{31}$$

yields

$$\frac{\omega^2}{k^2} = \frac{n}{M}\mathcal{V}(\mathbf{k}) = c^2, \tag{32}$$

where $\mathcal{V}(\mathbf{k})$ is the Fourier transform of V. Then follows an elegant argument (with my slight modifications) which narrows down possibilities for chosing the defecton interaction potential V. Temperatures at which the resonances were observed correspond to the frequency of the thermally populated modes: higher frequencies correspond to higher temperatures. On the other hand, concentration $n(T)$ from (27) is an increasing function of temperature. We therefore conclude:

$$\left(\frac{\partial n}{\partial w}\right)_c > 0. \tag{33}$$

But, from (32),

$$\left(\frac{\partial n}{\partial w}\right)_c = -\frac{Mc^2}{\mathcal{V}^2}\left(\frac{\partial \mathcal{V}}{\partial w}\right)_c. \tag{34}$$

As a result,

$$\left(\frac{\partial \mathcal{V}}{\partial w}\right)_c < 0 \Rightarrow \left(\frac{\partial \mathcal{V}}{\partial k}\right)_c < 0. \tag{35}$$

(Also $\mathcal{V}(\mathbf{k}) > 0$ from (32).) Out of plausible potentials of the form $1/r, 1/r^2, 1/r^3$, only $1/r^2$ satisfies the constraint (35). We obtain then:

$$\mathcal{V}(\mathbf{k}) = 4\pi V_1 a^2 \left(\frac{\pi}{2k} - a\right), \tag{36}$$

where V_1 is the value of $V(r)$ at the (lower) cut-off distance a. Aside from the line of argument, it is interesting to note that $1/r^2$ potential is also necessary to include in the wave-function from Chester's discussion (see Section 2.2 and [4], [6]), in order to account for the zero-point motion of the density waves in He II , which I did not mention at the time for the sake of clarity. Coincidence?

Wrapping up, resonance condition (24) with the help of (32) reads:

$$v^2 = \frac{n(T_R)}{M} 4\pi V_1 a^2 \left(\frac{\pi}{2k} - a\right). \tag{37}$$

The peak positions $T_R(30MHz) = .400K$ and $T_R(50MHz) = .765K$ allow to solve for unknown V_0 and a. However, in order for the obtained equations to have solutions, the authors in ([7]) had to assume that density of the defectons at zero temperature was finite, that is they modify (27) to read:

$$n(T) = \left(\frac{Mk_B}{2\pi\hbar^2}\right)^{3/2} \left[\mathcal{T}^{3/2} + T^{3/2} F_{3/2}\left(\frac{\Phi}{T}\right)\right], \tag{38}$$

Assuming a is on the order of lattice spacing, one obtaines $\mathcal{T} \sim 0.1K^{3/2}$. In the square brackets in Eq. (38), the two terms correspond to the condensed and thermally activated defectons, which gives then an estimate for the critical temperature $T_c \sim 0.1K$, if one assumes that the defecton gas is an ideal Bose gas, which is inconsistent with the value of a taken to be $2\mathring{A}$ in the paper. At least, $.1K$ can serve as an upper estimate for T_c.

I conclude the discussion of the ultrasonic attenuation results by mentioning that there are some recent results by Bijsma and Stoof [12] who derived the phenomenological traveling wave equations of Lengua and Goodkind with some sophisticated machinery, starting from a hamiltonian for interaction of point defects with the elastic strain in a crystal and arriving at a mode obeying superfluid dynamics and interacting with the sound waves according to Eqns.(22). They also propose a phase diagram which I give as Fig. 3.

4 Conclusions

In summary, I reviewed some of the theoretical and experimental arguments in favor of the existence of the so called supersolid state of matter. None of them is totally convincing, that way each one deserved to be discussed. I admit that I gave a very naive rendition of the ideas above, but it is good enough as a starting point. On a more concrete side, I totally dropped out the discussion of other possible but, in my opinion, less important for our discussion defects in solid ^{4}He, such as dislocations. [1] and [12] contain relevant references, as well as those to a large body of simulations indicating that it is worth to continue the search for a supersolid. However, all the simulations are confined to human-made models, and their relation to real life-system in this case is not at all obvious, at least not untill more convincing experimental results are obtained. It is quite plausible however that BEC of some kind of elementary excitations in solid ^{4}He does occur, but its consequences for the emergence of superfluidity are not clear. I should also mention some recent developments on a different kind of coherent excitations which might take place in solid ^{4}He, but in the bcc phase [8]. Those are coherent oscillations of local electrical dipoles, which arise due to local atom motions. Perhaps, something like those oscillations is responsible for anomalies observed by Lengua et al.

One thing surprised me while I was reading through the literature. All the efforts described above were concentrating on crystalline solids. How about amorphous solids? Those present a good example of a system having a finite residual entropy at zero temperature and finite number degrees of freedom at $T = 0$, as follows from their anomalously high specific heat down to lowest temperatures that have been tried. It might happen that the normal component of He II forms a glassy network before it crystalizes at higher pressures. Then we have a coexistence of a solid and a superliquid in one system. An experiment to detect such a glassy network would be difficult. An ice differs from the corresponding water by a non-zero sheer modulus and an enormously high (infinite) viscosity. Both would be very hard to check experimentally, because in the region around the melting curve, the critical velocity is very small.

References

[1] M.Meisel, Physica B 178 (1992) 121.

[2] O.Penrose and L.Onsager, Phys. Rev. 104 (1956) 576

[3] A.Andreev and I.Lifshitz, Sov. Phys. JETP 29 (1969) 1107

[4] G. Chester, Phys. Rev. A 2 (1970) 256

[5] A.Leggett, Phys. Rev. Lett. 25 (1970) 1543, J. Stat. Phys. 93 (1998) 927

[6] R. Feynman Phys. Rev. 91 (1953) 1301, *ibid* 94 (1954) 262

[7] G.Lengua and J.Goodkind, J. of Low Temp. Phys. 79 (1990) 251

[8] G. Nir, cond-mat/9904249

[9] De Boer, Physica, 14 (1948) 139

[10] L. Landau, I. Lifshits, vol.3 "Quantum Mechanics"

[11] L. Landau, I. Lifshits, vol.5 "Statistical Physics"

[12] M.Bijsma and H.Stoof, Phys. Rev. B 56 (1997) 14631, also cond-mat/9705307

Table 1
Summary of experimental searches for supersolid ^{4}He.

Group	Method	^{3}He/^{4}He concentration	Temperature range	Pressure range	Conclusion
Andreev et al. (1969) [23]	Plastic flow	Unstated	$T_{min} = 500$ mK	25–154 bar	$\frac{\rho_s}{\rho} < 1 \times 10^{-3}$ $v_c < 2 \times 10^{-7}$ cm/s
Suzuki (1973) [24]	Plastic flow	Unstated	1.38–2.12 K	29–51 bar	$v_c < 10^{-4}$ cm/s
Tsymbalenko (1976) [25]	Plastic flow	Unstated	0.6–2.1 K	25–40 bar	$v_c < 5 \times 10^{-10}$ cm/s
Dyumin et al. (1989) [28]	Plastic flow	Unstated	1.0–1.4 K	On the melting curve	$v_c < 10^{-9}$ cm/s
Greywall (1977) [26]	ΔP across weak-link	Unstated	$T_{min} = 30$ mK	25–50 bar	$\frac{\rho_s}{\rho} v_c < 2.5 \times 10^{-9}$ cm/s
Bishop et al. (1981) [27]	Torsional oscillator	$0.3–411 \times 10^{-6}$	$T_{min} = 25$ mK	25–48 bar	$\frac{\rho_s}{\rho} < 5 \times 10^{-6}$ $v_c < 5 \times 10^{-8}$ cm/s
Bonfait et al. (1989) [29]	Cylindric "U"-tube	Standard commercial ^{4}He	$T_{min} = 4$ mK	On the melting curve	$\frac{\rho_s}{\rho} v_c < 1.8 \times 10^{-9}$ cm/s
Adams et al. (1990) [31]	$P_v(T)$	$\sim 10^{-6}$	$T_{min} = 1$ mK	26 bar	$\frac{\rho_s}{\rho} < 10^{-5}$
van de Haar et al. (1991) [32]	$P_v(T)$	$\sim 10^{-7}$	1.5–120 mK	On the melting curve and 26 bar	$\frac{\rho_s}{\rho} < 2 \times 10^{-5}$ at 26 bar $< 6 \times 10^{-7}$ on melting curve
Lengua and Goodkind (1990) [30]	Ultrasound	1.5×10^{-9}	30 mK–1.0 K	~25 bar	$T_c = 117$ mK(?) $\frac{\rho_s}{\rho} \sim 10^{-3}$(?)

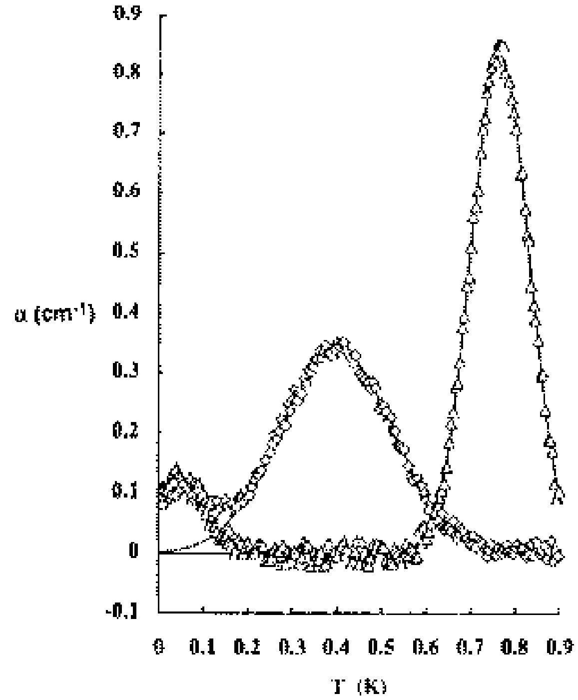

Fig. 1 Resonant portion of the attenuation data at 30 and 50 MHz (diamonds and triangles, respectively). The data has been decimated for clarity.

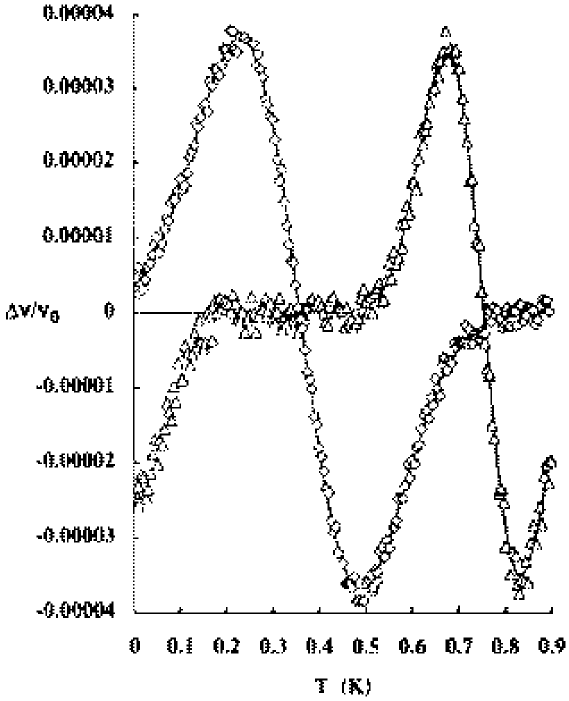

Fig. 2 Resonant portion of the velocity data at 30 and 50 MHz (diamonds and triangles, respectively). The data has been decimated for clarity.

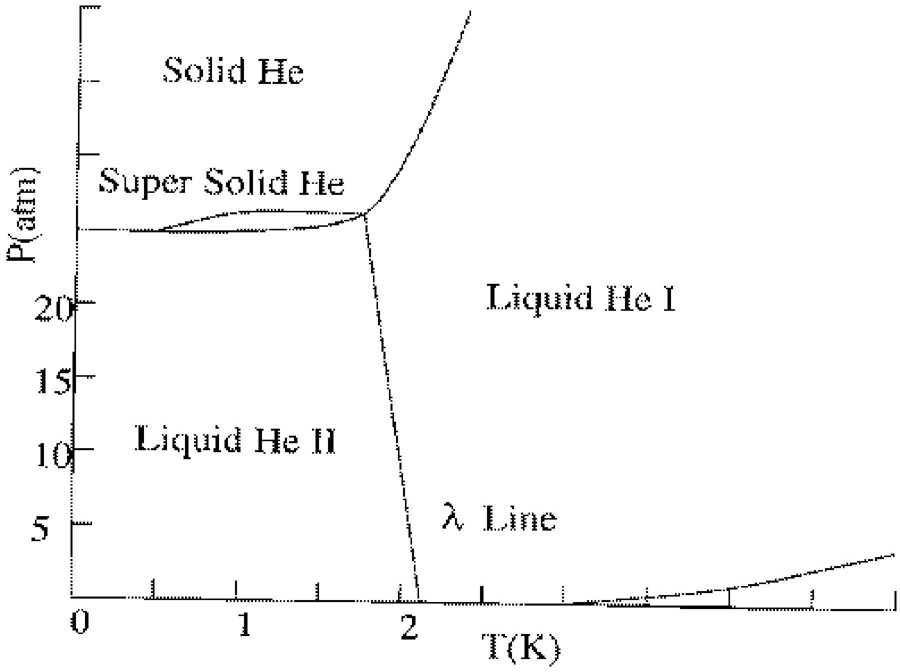

Fig. 3