Effects of Nodes on specific Heat in High T_c Superconductors

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Abstract

Specific heat measurements attracted many researchers by its simple criterion for order parameter of High T_c Superconductors(HTS). The nodal effects on HTS are confirmed: (i) $C_{electron} \propto T^2$ at $T \ll T_c$ in a zero field. (ii) $C \sim TB^{1/2}$ with $\mathbf{B} \| \mathbf{c}$-axis. But, there is still a controversy about where $C/TB^{1/2} =$ constant is valid. In a geometry of $\mathbf{B} \perp \mathbf{c}$-axis, four-fold symmetry with field directional dependence is predicted as an evidence of line nodes in order parametr.

1 Introduction

Due to nodes in momentum space, many characteristic features occur in a High T_c superconductor. Experiments such as magnetic penetration depth, angle resolved spectroscopy, etc., took advantage of the nodal effects in probing order parameter in HTS. A growing consensus of the d-wave pairing in the cuprates is coming out from those experiments[1]. But the consensus supported by the surface sensitive experiments should be also checked by bulk measurements. Specific heat measurement can give a simple criterion for order parameter. Although it is insensitive to the phase of the order parameter, it can provide bulk information on the behavior of the density of states[DOS, $N(E)$] near the Fermi level E_F. The electronic specific heat of a

s-wave superconductor is exponentially activated as $exp(-\Delta_0/k_BT)$, where Δ_0 is a minimum gap. In contrast, a superconductor with lines of nodes has a contribution from low-lying excitation even at low temperatures. The d-wave density of states (DOS) rises linearly with energy at Fermi level, $N(E - E_F, B = 0) \propto E$. It results in an electronic specific heat of T^2 ($Fig.1$). In the vortex state, Volovik[2] calculated the effect of nodes onto specific heat by considering Doppler shift from superflow around a vortex. As for a d-wave superconductor which has line nodes,

$$C(T,B) \sim N_F T_c \frac{B}{B_{c2}} \frac{x}{1+x},$$

$$x \sim \frac{T}{k_F v_s} \sim \frac{T}{Tc}(\frac{B_{c2}}{B})^{1/2}$$

Here, N_F is the DOS at the Fermi level in the normal metal, B_{c2} is an upper critical field at $T = 0$, and v_s is the characteristic value of the superfluid velocity in the vortex array, given by $v_s \sim \hbar/mR$, R is intervortex distance. For the superflow-dominated regime, $x \ll 1$, $C \sim TB^{1/2}$ and for the temperature dominated regime, $x \gg 1$, $C \sim B$ with no temperature dependence is expected.

2 Specific Heat with B $\|$ c-axis

Compared to the simple prediction, the data analysis is very complicated due to background subtraction. The extrinsic magnetic contributions due to weakly interacting paramagnetic centers give sample-dependent low-T upturn[3] and the lattice contribution is very hard to identify because the T^3 term may be contaminated by electronic component. To detect the minor contribution, $C_{electron}/T \sim T$, Momono et al. used indirect technique of differential measurements versus doping[4], whereas fits of the total specific heat give more ambiguous results[5,6]. Other groups investigated the variation of specific heat versus magnetic field[5,6]. Due to the Doppler effect of charge carriers from vortices, there is notable contributions from the vicinity of the nodes, which is $Cvortex/T \sim B^{1/2}$. But, this term is still a small fraction of the total specific heat. Besides, various species of paramagnetic centers add their own variation with B even though the lattice contribution is independent of B.

Revaz et al.[7] used anisotropy of the specific heat in a magnetic field $C_{aniso}(T, \mathbf{B}) = C(T, \mathbf{B} \parallel \mathbf{c}) - C(T, \mathbf{B} \perp \mathbf{c})$. The lattice specific heat, the zero field d-wave term $C_{electron}(T,0)/T \propto T$, the nuclear specific heat, other paramagnetic contributions, and possible residual normal-state contributions are claimed to be canceled out. Here $C_{aniso}(T,B)$ essentially represents $C_{vortex}(T,B)$ because the specific heats in both geometry of in-plane and out of plane is consistent within experimental error. Scaling formula of the d-wave form with taking into account of the uniaxial anisotropy is used:

$$\frac{Caniso(T,B)}{\gamma\, T}(\frac{B_{c2}}{B})^{1/2} = k\{f(x) - \frac{f(\Gamma^{1/2}x)}{\Gamma^{1/2}}\} \equiv k f_{\Gamma}(x),$$

where γ is Sommerfeld constant of order 1, B_{c2} is the upper critical field for $B \parallel c$, k is a dimensionless constant of order 1, and Γ is the anisotropy factor of the upper critical field, which is $(m_c/m_{ab})^{1/2} = 5.3 \pm 0.5$ for the present overdoped crystal of $YBa_2Cu_3O_{7.00}$. The empirically interpolated function $f(x)$ is $(1+x)^{-1}$ and $x = \alpha T B^{-1/2}$,where α is a prefactor of order $B_{c2}^{1/2}/T_c$. The general scaling relation $C_{aniso}/TB_{1/2} \propto f_{\Gamma}(x)$ is verified($Fig.2$), which is an evidence of the presence of the line nodes. But the data for $1.2 < T < 10K$ do not lie in the $x \ll 1$ regime because C_{aniso}/T varies with T even in highest field. Neither in the regime of $x \gg 1$,where linearity in B is predicted.

In a recent paper[8], D.A. Wright et al. reported that $C_{vortex} \sim B^{1/2}T$ at low temperature , but with a crossover at a value $x_c \sim B_{c2}^{-1/2}T_c$ of the parameter $x \sim B^{-1/2}T$ to a high -T, low -B region in which both a T^2 term that is independent of B, and a T-independent B-proportional term appeared as predicted($Fig.3$). The data showed the crossover at $x_c \sim 6.5T^{-1/2}K$ in the sample of $YBa_2Cu_3O_{7-\delta}$, $0 \leq \delta \leq 0.2$. In Revaz data, he suggested that the true asymptotic behavior $C_{vortex}/B^{1/2}T = const$ can be reached only below $x \approx 0.2T^{-1/2}K$, e.g., at $T < 0.6K$ in $B = 8T$ and still lower temperatures in lower fields while Wright data showed apparent asymptotic behavior below $x_c(Fig.4)$. Even though all the above mentioned experiments showed the evidence of the presence of line nodes, the apparently different dependence of the mixed state C_{vortex} on both H and T has to be solved.

3 Specific Heat with B $\perp$ c-axis

Salamon et al.[9,10] measured thermal conductivity(κ) of untwined single crystal of $YBa_2Cu_3O_{7-x}$ with a field of $1.5T$ applied to CuO_2 planes at $10K$. They found the four-fold field directional dependence of κ and concluded that it is the evidence of the existence of nodal lines in the gap function($Fig.5$). The specific heat measurement in this geometry is almost an uncharted area. Most researchers assumed that there is no special feature in this geometry and used the $C(\mathbf{B} \parallel$ a-b planes) as a background subtraction. But, to consider the thermal conductivity measurement by Salamon et. al., it is naturally expected that we could see the four-fold symmetry in specific heat. Recently, Vekhter et al.[11] proposed a theory that predict the symmetry in specific heat. In the estimation of density of states, they assumed: (i) The high T_c cuprates are strongly anisotropic, with the CuO_2 planes separated by a larger distance than the c-axis coherence length. Since c-axis transport is incoherent, only the energies of the quasiparticles with momenta in the plane are responsible for the thermodynamic properties and should be Doppler shifted. (ii) Even though the structure of the vortex core for $\mathbf{B}$ in the plane significantly differs from that of a conventional Abrikosov vortex, the structure of the vortex state outside the non-linear core is very similar to that of an Abrikosov flux lattice.

When a magnetic field $\mathbf{B}$ applied in the a-b plane at an angle α to the x-axis, Doppler shift occurs to the extended quasiparticle states by $\delta w(\mathbf{r}) = \mathbf{v}_s \cdot \mathbf{k}$. The pure d-wave gap function is $\Delta_k = \Delta_0 \cos 2\phi$. If we approximate the unit cell of the vortex lattice by a circle of radius R, we obtain

$$\delta w_k(\mathbf{r}) \quad = \frac{E_B}{\varrho} sin\beta sin(\phi - \alpha),$$

where $\varrho = r/R$ and E_B is the energy scale associated with the Doppler shift

$$E_B = \frac{av^*}{2}\left(\frac{\pi B}{\Phi_0}\right)^{1/2} = v\Delta_0.$$

Here, a is a geometric constant of order unity, and in the London theory the rescaled Fermi velocity $v^* = v_F(\lambda_{ab}/\lambda_c)^{1/2}$, where v_F is the Fermi velocity in the plane.

For the field applied perpendicular to the CuO_2 layers, the average Doppler shift is constant on the Fermi surface because the momenta of all quasiparticles(QPs) are normal to $\mathbf{B}$. But, for the field in plane, the average Doppler

shift becomes dependent on the position at the Fermi surface as wee see $E_B \sin(\phi - \alpha)$ in δw . Since QPs contribute to the DOS if their Doppler shifted energy overcomes the local energy gap, it is expected that the DOS depends on the angle between the applied field and the direction of nodes of the order parameter.

By taking advantage of the fact that the DOS is dominated by the contribution of the regions of the Fermi surface near the gap nodes, we get DOS at Fermi level for $T, E_B \ll \Delta_0$,

$$N_0(\alpha) \tilde{} \frac{E_B}{\pi \Delta_0} \sum_{nodes} \mid E_B \ \sin(\phi - \alpha) \mid = 2\sqrt{2} \frac{E_B}{\pi \Delta_0} \max(|\sin \alpha|, |cos.\alpha|)$$

The DOS shows four-fold oscillations as a function of the angle of the applied field($Fig.6$). For a field in the nodal direction, two of the nodes do not contribute to DOS, while the contribution of the other two is a factor of $\sqrt{2}$ larger than it was for the field along an anti-node($Fig.7$). The overall density of states is therefore reduced by 30%. They reported 8% of reduction in the amplitude of oscillations when they calculated numerically in 3-d superconductors. So, it is expected that in realistic materials, the amplitude of the oscillation is somewhere between the two estimates and well within the experimental resolution of the specific heat measurements. Other than the four-fold symmetry, they also predicted that the temperature dependence of $C/TB^{1/2}$ is linear for the field in a nodal direction, and quadratic for $\mathbf{B}$ away from the node. To my knowledge, there has yet to be an experimental report that can confirm those claims.

Note: Our group made a new probe designed for this research. I personally hope that our experiment can give an insight on those claims. And I'd like express my gratitude to the instructor, proff. Nigel Goldenfeld, for giving me a chance to write down this term paper.

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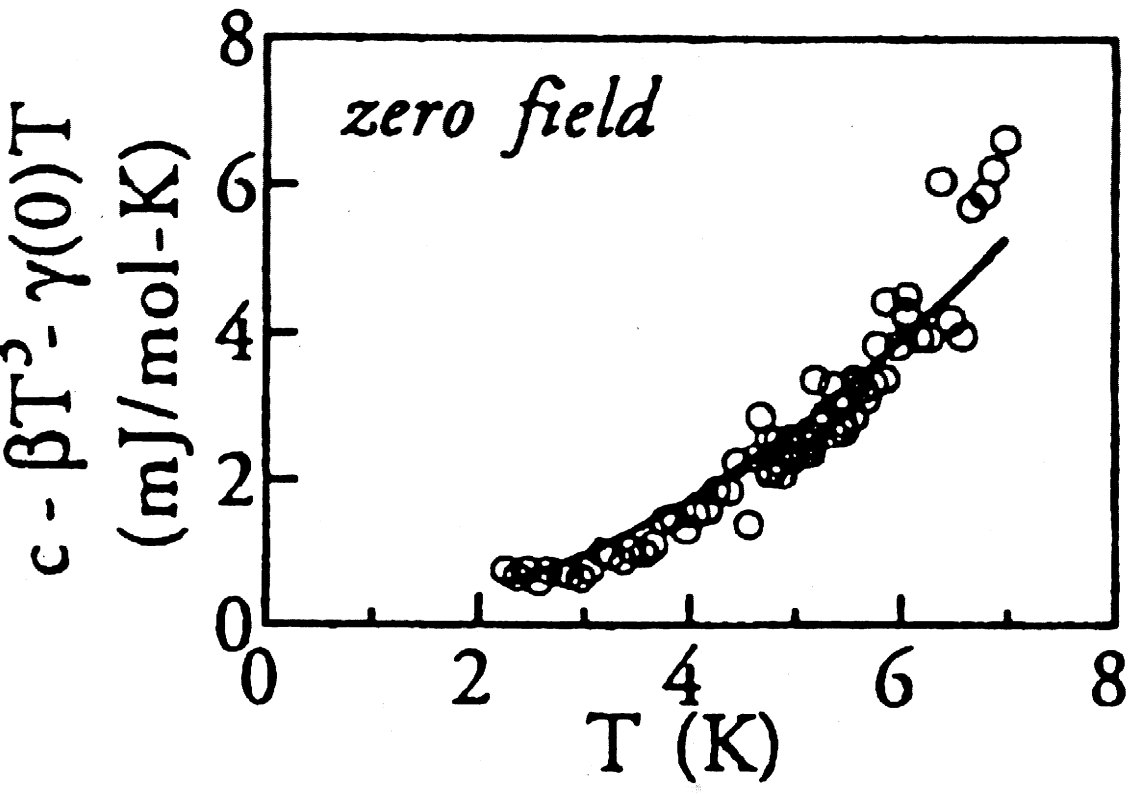

Figure 1: Zero-field electronic specific heat $C_{electronic}$ vs T. The solid line is a fit by αT^2 with $\alpha = 0.11 mJ/molK^3$(by Moler et al.[5])

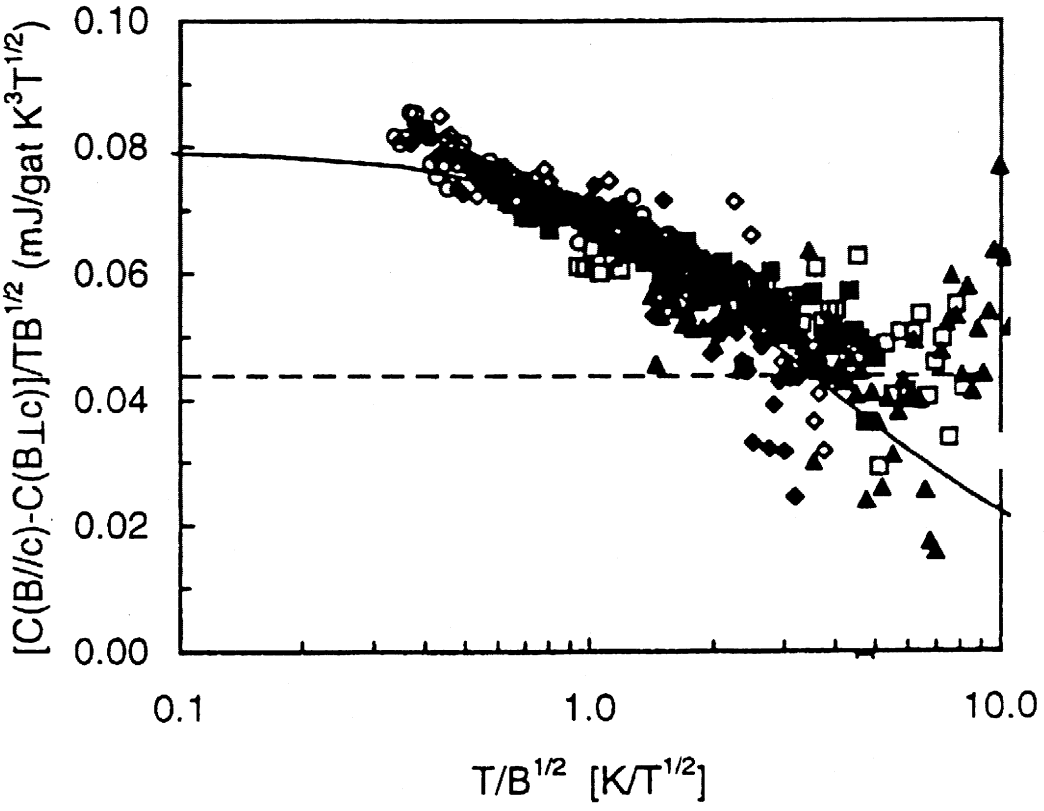

Figure 2: Scaling plot of the anisotropic specific heat $C_{aniso}/TB^{1/2}$ ($1.2 \leq T \leq 10K, 1 \leq B \leq 14T$). Full line, scaling function $f_\Gamma(x)$ based on $f_\Gamma(x) = (1+x)^{-1}$ was the fitted parameters. Dashed line, function used in Ref[5], $C(T, B \perp c)/TB^{1/2} = 0.91 mJ/molK^2T^{1/2}$(by Revaz et al.[7])

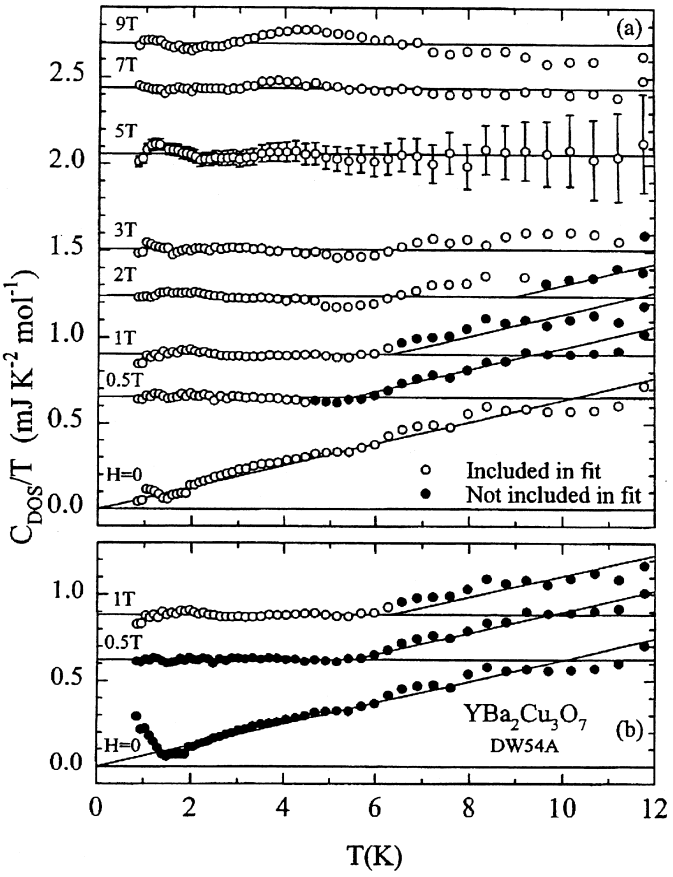

Figure 3: C_{DOS} as obtained in two fits in which different points, shown as solid circles, were omitted: (a) the basic fit; (b) a fit with all 0 and 0.5T data omitted. All lins that are not horizontal have the same slope, that of the fit to the $H = 0$ data in (a). The error bars on the 5T points represent $\pm 0.5\% C(H)/T$. They would be approximately the same for all H(by Wright et al.[8])

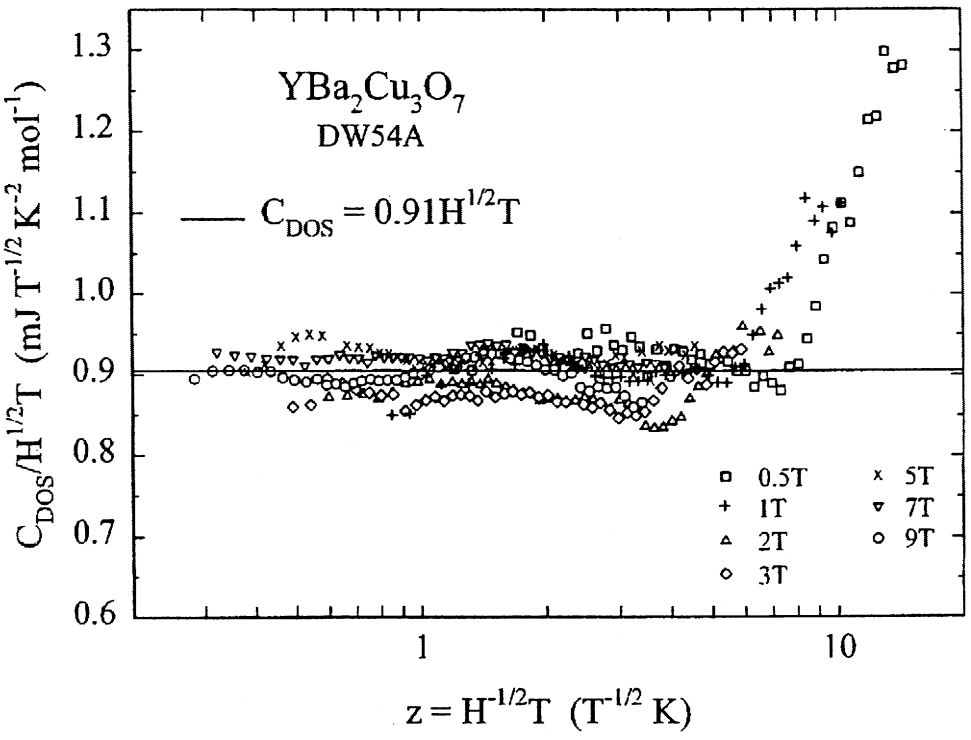

Figure 4: Scaling of $C_{DOS}/H^{1/2}T$ vs x(by Wright et al.[8]

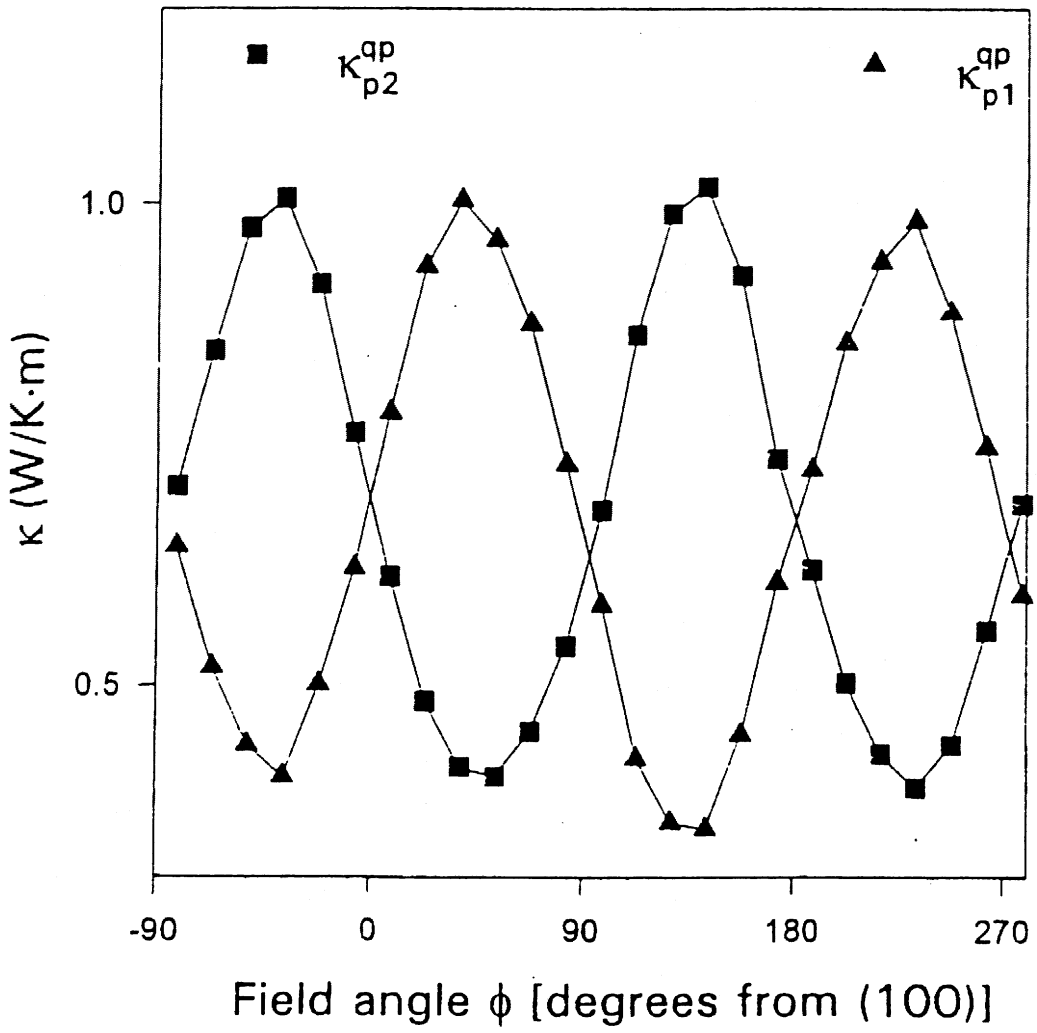

Figure 5: Nominal thermal conductivities $\kappa_1(circles)$ and $\kappa_2(squares)$ determined from $\Delta\ T_1$ and $\Delta\ T_2$, respectively, vs the field angle ϕ(by F. Yu et al.[10]).

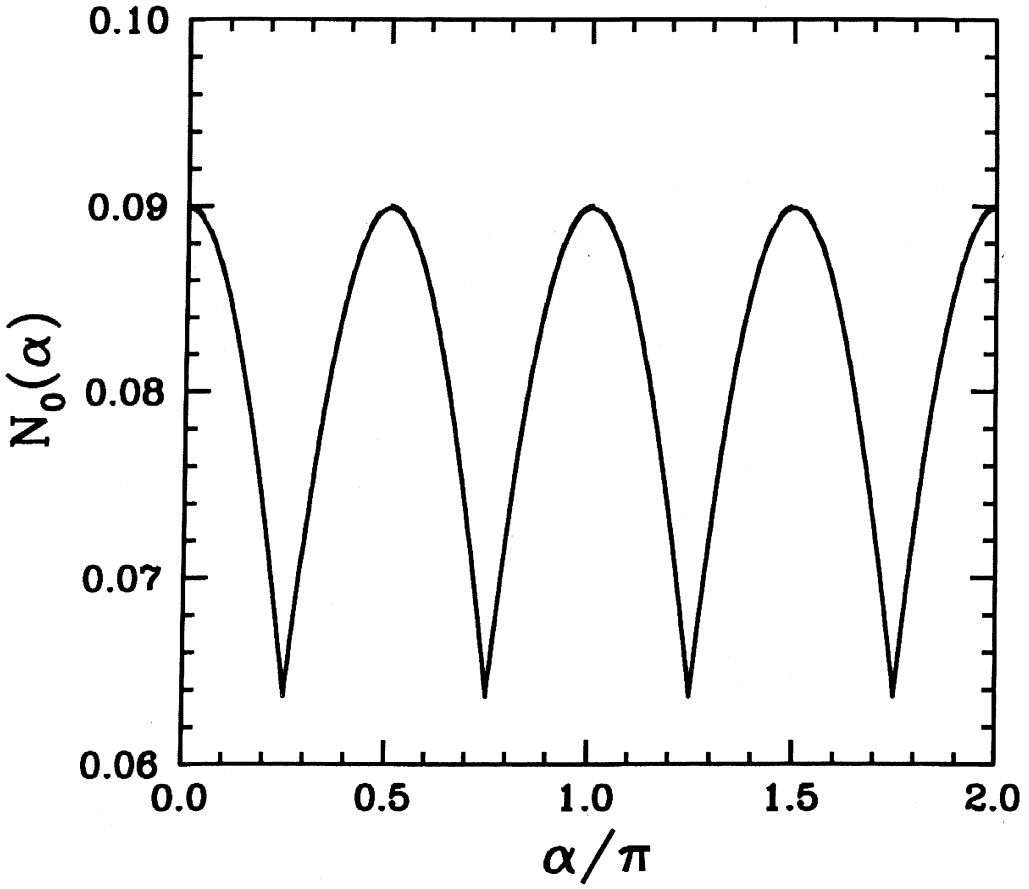

Figure 6: The density of states at the Fermi level, $N_0(\alpha)$, for $E_B/\Delta_0 = 0.1$(by Vekhter et al.[11])

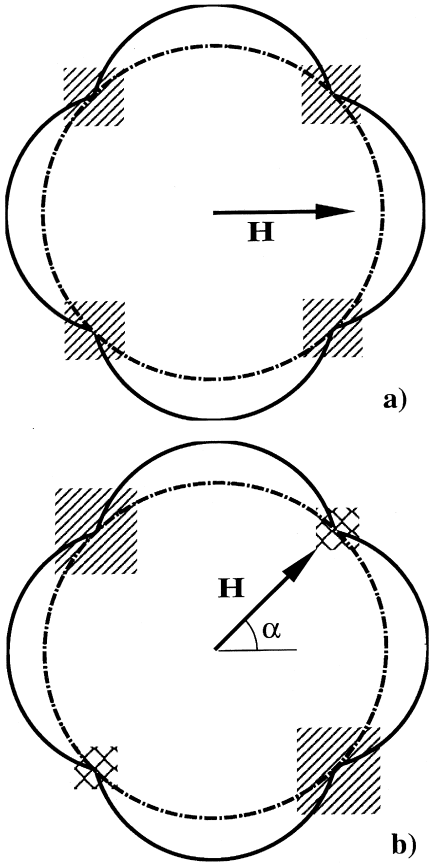

Figure 7: The regions contributing to the density of states for the anti-nodal(a), and nodal(b) orientation of the magnetic field(by Vekhter et al.[11])