Phase Transition in the Macromolecules

Kai Xiu

May 4 1998

It is well known that the functioning of biopolymers and primarily of proteins in a living cell is determined by a unique spatial ternary structure, this structure is called protein globule. It is also well known that when the protein is heated its denaturation occurs, the globular spatial structure is destroyed and the biochemical activity ceases. In this case one says that the globule is transformed into the coil. Similar phenomena can also be observed in simple non-biological polymers. As a definition, we shall call a a globule the state of a macromolecule in which it has a definite thermodynamically reliable spatial structure, whose density fluctuations are less than the density itself, or the range of the density correlations is finite when $N \to \infty$. In contrast, a coil is the state of a macromolecule without any spatial structure. The coil pulsates macroscopically, the density fluctuations in it are of the same order as the density itself, and the radius of correlation is of the same order as the dimensions of the coil, i.e., it tends to infinity when $N \to \infty$. Thus, according to our definition, the globule and the coil have different fluctuation regimes. From the point of view of statistical physics it is natural to call such significantly different states the phases, and the transition between them the phase transition.

The statistical physics of macromolecules as a branch of science appeared in the 1930s, after H. Staudinger demonstrated experimentally in 1922 a chain structure of polymer molecules. However, before late 1960s this area of research remained rather detached and isolated, being sooner regarded as belonging to physical chemistry than to condensed matter physics. The situation dramatically changed during the late 1960s, when the polymer theory attracted the attention of some leading theoretic physicists, I. M. Lifshitz in the USSR, S. F. Edwards in the UK and P. G. De Gennes in France. Under the talented work of them, people began to make clear the formation of a system of simple models and qualitative concepts about the fundamental physical properties of polymers on the molecular level, and put the successful utilization of these concepts in both the physical chemistry of polymers and molecular biophysics.

A simple polymer model can be describe as a freely jointed chain, composed of a sequence of N rigid segments, each of length l and able to point in any direction independently of each other.

First let us consider the case when there is no interaction between segments which are not chemically bonded. To characterize the chain conformation, we consider the end-to-end vector $\mathbf{R}$ and calculate the mean square $\langle \mathbf{R}^2 \rangle$ by averaging over all the possible conformations. It is easy to show that

$$\langle \mathbf{R}^2 \rangle = Nl^2 \tag{1}$$

thus for a multi-link chain $N >> 1$, the mean size of a macromolecule $R = \langle \mathbf{R}^2 \rangle \sim N^{1/2} l$ is much less than the total length Nl measured along the contour of the polymer chain.

In addition to the mean square of the end-to-end vector $\langle \mathbf{R}^2 \rangle$ of the coiled chain characterizing the mean size of the coil, there is a more specific quantity describing the coil, $P_N(R)$, i.e., the probability distribution function that the end-to-end vector of the chain consisting of N links equals R. By using the central limit theorem of probability theory, such a quantity has the Gaussian distribution

$$\begin{aligned} P_N(R) &= \left(\frac{2\pi N l^2}{3}\right)^{-3/2} exp\left(-\frac{3R^2}{2Nl^2}\right) \\ &= \left(\frac{2\pi \langle R^2 \rangle}{3}\right)^{-3/2} exp\left(-\frac{3R^2}{2\langle R^2 \rangle}\right) \end{aligned} \tag{2}$$

We can observe that both the mean square $\langle \mathbf{R}^2 \rangle$ of the Gaussian coil and the probability distribution $P_N(R)$ depend on only one quantity specified by the chemical structure of a specific polymer: the Kuhn segment length l.

If we denote $P(x_1, x_2, x_3, \ldots) = P_N\{x_l\}$ as the probability density of the configuration of a specific chosen set of coordinates, it takes the form

$$P(x_1, x_2, x_3, \ldots) = g(x_1, x_2) g(x_2, x_3) \cdots g(x_{N-1}, x_N) \tag{3}$$

here $g(x_t, x_{t+1})$ is the conditional probability of finding the end of the additional section at the point x_{t+1}, provided its beginning (i.e., the end of the foregoing section) is fixed at the point x_t. Systems with the probability distribution of type (3) are well known as Markov chains. We see that an ideal polymer chain thus belongs to this class.

To deal with the partition function of a polymer chain, we start from the distribution function $\rho(\Gamma)$, i.e.

$$\rho(\Gamma) = g(x_0, x_1)g(x_1, x_2)\cdots g(x_{N-1}, x_N) \tag{4}$$

here the microscopic state Γ is denoted as

$$\Gamma = \{x_0, x_1, \cdots, x_N\} \tag{5}$$

the partition function is obtained by integrating the distribution $\rho(\Gamma)$. It is often convenient to consider a macromolecule whose both terminal links are fixed in space, that is, with the links indexed 0 and N fixed at the given points x_0 and x_N, respectively. The partition function of such a macromolecule depends on x_0 and x_N and is called the Green function of the polymer chain, or the propagator. It is designated as

$$G(0 : x_0; N : x_N) \equiv G_N(x_N) \equiv \int \rho(\Gamma')\delta(x'_0 - x_0)\delta(x'_N - x_N)dx'_0 dx'_1 \cdots dx'_N \tag{6}$$

By definition, the Green function (6) can be written for the simplest case as

$$G_N(x_N) = \int g(x_0, x_1)g(x_1, x_2)\cdots g(x_{N-1}, x_N)dx_1 dx_2 \cdots dx_{N-1} \tag{7}$$

It is easy to see (7) has a recursive relation

$$G_{N+1}(x_{N+1}) = \int G_N(x_N)g(x_N - x_{N+1})dx_N \tag{8}$$

After introducing the operator

$$\hat{g} : \qquad \hat{g}\psi(x) = \int g(x, x')\psi(x')dx' \tag{9}$$

Eq (7) can be rewritten in the form

$$G_{N+1}(x) = \hat{g}G_N \tag{10}$$

Now let us introduce the notion of effective external fields, in term of which many effects experienced by polymer systems can be phenomenologically described. The essence of effective external fields does not mean a real physical field, it is only a way to describe a spatial inhomogeneity or anisotropy of the external conditions or to express the dependence of the energy φ of link on generalized coordinates. Thus, the compressing external field in which the energy of the link depends on its spatial position $\varphi(x)$ may characterize an attraction of the links to a foreign particle suspended in a solvent, their adsorption on a surface separating solvent phases, the placing of the chain into a microscopic cavity of limited volume, and so on.

Bearing this in mind, the revised distribution function $\rho(\Gamma)$ has the form

$$\begin{aligned} \rho(\Gamma) = \quad & exp(-\varphi(x_0)/T)g(x_0, x_1)exp(-\varphi(x_1)/T)g(x_1, x_2) \\ & \cdots exp(-\varphi(x_{N-1})/T)g(x_{N-1}, x_N)exp(-\varphi(x_N)/T) \end{aligned} \tag{11}$$

in this case, the new Green function obeys a similar equation

$$G_{N+1}(x_{N+1}) = \hat{Q}G_N(x_N) \tag{12}$$

such a type of relation plays a role of diffusion equation and, in many cases, reduces to a common differential equation. Its formal solution can be written in the form

$$\begin{aligned} G(0 : x_0; N : x_N) &= [\hat{Q}]^N(x_0, x_N) \qquad (13) \\ &= \int Q(x_0, x_1)Q(x_1, x_2)\cdots Q(x_{N-1}, x_N)d^3x_1 d^3x_2 \cdots d^3x_{N-1} \end{aligned}$$

The Green function is also convenient written in the form of a bilinear series. We define the eigenfunctions and eigenvalues of $\hat{Q}$ as

$$\begin{aligned} \hat{Q}\psi_m &\equiv \int \hat{Q}(x', x)\psi_m(x')d^3x' = \Lambda_m\psi_m(x) \\ \hat{Q}^\dagger\psi_m^\dagger &\equiv \int \hat{Q}(x', x)\psi_m^\dagger(x')d^3x' = \Lambda_m\psi_m^\dagger(x) \end{aligned} \tag{14}$$

in the proper basis, $\hat{Q}$ is diagonal, that is

$$Q(x', x) = \sum_m \Lambda_m\psi_m^\dagger(x')\psi_m(x) \tag{15}$$

as a result, we obtain the form of Green function as

$$G(0 : x_0; N : x_N) = \sum_m \Lambda_m^N \psi_m^\dagger(x_0)\psi_m(x_N) \tag{16}$$

If the spectrum is discrete, then the largest eigenvalue Λ is separated from the neighboring one by a gap, as $T \to \infty$, the corresponding term of the series (16) becomes much larger than all the others

$$G(0 : x_0; N : x_N) = \Lambda^N \psi_0^\dagger(x_0)\psi_0(x_N) \tag{17}$$

The partition function of the chain derived from the Green function by summing over the states of the ends, is

$$Z_N = const \quad \Lambda^N \tag{18}$$

and the corresponding free energy

$$\begin{aligned} F_N &= -T \ln Z_N = -TN \ln \Lambda = N\lambda & (19) \\ \Lambda &= exp(-\lambda/T) & (20) \end{aligned}$$

since

$$E_N = \int \varphi(x) n(x) d^3x \tag{21}$$

we get

$$S = \int \frac{\varphi(x) - \lambda}{T} n(x) d^3x = \int n(x) \frac{\hat{g}\psi}{\psi} d^3x \tag{22}$$

It is very instructive to see the outcome of the expression of the entropy when assuming $n(x)$ varies smoothly, then $\hat{g}$ can be substituted by $1 - \frac{a^2}{6}\nabla^2$, and $\hat{\psi}(x)$ by $n^{1/2}(x)$, that is

$$\begin{aligned} S\{n\} &= \frac{a^2}{6} \int n^{1/2}(x) \nabla^2 n^{1/2}(x) d^3x \\ &= const - \frac{a^2}{6} \int \left[\nabla n^{1/2}(x)\right]^2 d^3x & (23) \end{aligned}$$

As expected, the local concentration gradient is unfavorable in terms of entropy. This effect is caused by the link connectivity that is formally manifested in $S\{n\}$ being proportional to a^2, the value characterizing chain bonding.

Till now we have studied the thermodynamic form of the polymers in terms of a freely joint chain, in which no interaction are between the non-adjacent links. Obviously, monomers which are distant from each other along the chain can approach and touch each other in space. In this case the forces of volume interaction are acting between these monomers. This forces differ essentially from the interactions between adjacent segments. Interaction between monomers which are distant from each other along the chain are essentially the same as those acting between the atoms of a real gas, which involve strong short range repulsions and long-range attractions between chain sections, as shown in Figure (1). On small length scales, repulsion is predominant ($\frac{\partial u}{\partial r} < 0$) because of the geometric volume of links, while at longer distance, the links are attracted by Van der Waals forces ($\frac{\partial u}{\partial r} > 0$). At a moderate concentration, the volume interactions are reduced to relatively rare link collisions and are described by *viral* coefficients of the disconnected link system, which is in a similar way as the treatment of real gas

$$F = F_{id} + F_{int}, \qquad F_{int} = NT(nB + n^2C + \ldots), \tag{24}$$

$$p = nT(1 + nB + 2n^2C + \ldots) \tag{25}$$

where N is the number of particles in the system, F_{id} the part of the free energy corresponding to the ideal gas of N particles, F_{int} the volume interaction contribution to the free energy, and B and C the second and third *virial* coefficient respectively, in which B is defined as

$$B(T) = \frac{1}{2}\int \left(1 - e^{-u(r)/T}\right) d^3r \tag{26}$$

It is useful to evaluate B from Eq. (26) at different temperatures for the interaction potential shown in Fig. (1). At high temperature, the attractive part of the $u(r)$ is negligible, Eq. (26) yields $B \sim v$, where v is so called excluded volume of the particle. As the temperature lowers, the second term in the integrand of Eq. (26) becomes more important, the value of B diminishes tending to zero at a temperature θ linearly with the difference $T - \theta$ and turning negative for $T < \theta$:

$$B \sim \begin{cases} v\tau, & \tau \leq 1, \\ v, & \tau \geq 1, \end{cases} \qquad \tau \equiv \frac{T-\theta}{\theta} \tag{27}$$

As a result, the Gibbs distribution for a polymer system can be expressed as

$$\rho(\Gamma) = exp\left(-E(\Gamma)/T\right) \prod g_i \tag{28}$$

where g_i are the "bonds" and $E(\Gamma)$ the "interactions" which are irrelative of the nature of the polymer's links. $E(\{n\})$ (or $E(\Gamma)$) represents the volume interaction contribution and has a exact analogue of F_{int} in Eq. (24), which can be written as

$$E\{n\} = \int \left[f\left(n(x), T\right) - f_{id}\left(n(x), T\right)\right] d^3x = \int f^*\left(n(x), T\right) d^3x \tag{29}$$

where

$$f^*(n, T) = f(n, T) - f_{id}(n, T) \tag{30}$$

The physical meaning of Eq. (29) is quite clear. The contribution of volume interaction to the free energy of a polymer system is local, and it is determined by the free energy of a unit volume of the corresponding system of disconnected links less the quantity f_{id}, responsible for the translational entropy of mutually independent motion of the disconnected links, which is derived from the ideal gas

$$f_{id} = Tn\ln(n/e) \tag{31}$$

For the purpose to remind, this contribution is excluded, because the links connected in the polymer chain can not move freely.

The function $f^*(n, T)$ can not be calculated accurately in the whole range of variation of its arguments, although some kind of numerous treatment is possible(we will mention it later). We also write out the two other quantities to be required later, the contribution of volume interactions to the chemical potential of the link and to the pressure, respectively

$$\mu^* = \frac{\partial f^*(n, T)}{\partial n}, \qquad p^* = n\mu^* - f^* \tag{32}$$

The well known Van der Waals equation of state, describing the gas-liquid transition yields

$$f^* = -nTln(1 - nv) - n^2 a \tag{33}$$

$$\mu^* = -Tln(1 - nv) + \frac{Tnv}{1 - nv} - 2na \tag{34}$$

$$p^* = Tvn^2\left[\frac{1}{1 - nv} - \frac{a}{Tv}\right] \tag{35}$$

where v and a are parameters. In consistent with Eq. (33), (34), (35), we expand them in a series of power n

$$\begin{aligned} f^*/T &= n^2B + n^3C + \cdots && (36)\\ \mu^*/T &= 2nB + 3n^2c + \cdots && (37)\\ p^*/T &= n^2B + 2n^3C + \cdots && (38) \end{aligned}$$

compare them with Eq. (33), (34), (35), we get the *virial* coefficient B and C

$$B = v - a/T, \qquad C = v^2/2 \tag{39}$$

A typical dependence of the thermodynamic functions μ^* on n and T for the Eq. (37) is shown in Figure (2). Its behavior at low N is governed by the second *virial* coefficient B, unlimited growth of μ^*corresponds to the approximation of the dense packing of the particles. The range of concentrations appearing at lower temperatures, where $\mu^* < 0$ corresponds to a domination of attractive forces in particle interactions.

It is shown that the entropy $S\{n\} \sim N^\alpha$ where α is equal to 1/3 or 2/3, i.e., is always less than unity. At the same time, $E\{n\}$ is proportional to N. Thus for long enough chains the free energy is equal in a first approximation to

$$F\{n\} = E\{n\} - TS\{n\} \approx E\{n\} = \int f^*(n) d^3x \tag{40}$$

Denote n_0 as the one makes $f^*(n)$ minimum, we can proceed to the form

$$F\{n\} \simeq \nu f^*(n_0) = Nf^*(n_0)/n_0 \tag{41}$$

take the minimum of the free energy $F\{n\}$ and we get

$$\frac{\partial}{\partial n_0}\left(\frac{f^*(n_0)}{n_0}\right) = \frac{n_0\mu^*(n_0) - f^*(n_0)}{n_0^2} = \frac{p^*(n_0)}{n_0^2} = 0 \tag{42}$$

this shows

$$p^*(n_0) = 0 \tag{43}$$

is just the condition of the oncome of a stable globule out of state of coils. We can rewrite Eq (43) in a form

$$0 = -p^*(n_0) = f^*(n_0) - n_0\mu^*(n_0) = \int_o^{n_0} \left[\mu^*(n) - \mu^*(n_0)\right] dn \tag{44}$$

the geometric meaning of this condition is a coincidence of the shaded areas in Figure (2). One can see that the equilibrium density n_0 of the globule and the volume free energy $\mu^*(n_0)$ per link are easily found from this illustration.

Figure (2) shows the simplest and most typical variation of the dependence $\mu^*(n)$ with temperature. As temperature decreases, the second *virial* coefficient, defining the slope of the plot $\mu^*(n)$ at $n = 0$, becomes negative before all other coefficient. In other words, with the temperature decreasing, attraction begins to prevail over repulsion first in binary collisions. Specially, such a situation is described by the Van der Waals equations. In practical situation, on approach to the θ point from poor solvent, a globule gradually swells, its size becoming closer to that of a coil, as it should be on approach to the phase transition point. From the comparison of Eq. (41) and Figure (2), it can be seen that the volume contribution of free energy of the globule is negative, $F_{vol} < 0$, at a temperature below θ, which is the condition of the thermodynamic advantage of the globule state. Consequently, the transition temperature $T_{tr} = \theta$. Recall that near the θ point

$$B(T) \simeq b\tau, \qquad \tau = (t-\theta)/\theta, \qquad C(T) \simeq C = const > 0 \tag{45}$$

Moreover, we substitute the *virial* expansion Eq. (38) into the condition (43) and get

$$\begin{aligned} p^*(n_0) &\simeq TBn_0^2 + 2CTn_0^3 = 0, \quad n_0 = -B/2CorV = -2NC/B & (46) \\ \mu^* &= 2TBn + 3CTn^2 \simeq -B^2T/4C & (47) \end{aligned}$$

Using the formula we have just obtained for B and C we get

$$\begin{aligned} n_0 &\simeq |\tau|b/2C, \qquad R \sim N^{1/3}|\tau|^{-1/3}(C/b)^{1/3} & (48) \\ F_{vol} &= N\mu^*(n_o) \simeq -N\tau^2\theta b^2/4C \sim -N\tau^2\theta & (49) \end{aligned}$$

Hence, the result implies that the globule-coil transition is a second-order phase transition.

Till now, we did not take into account of the entropic contribution to the free energy. In fact, on approach to the θ-point, the thickness of the surface layer of the globule grows, and the volume approximation needs corrections in this region. To see why the surface plays such a important role on the entropic contribution to the free energy, let us recall the expression Eq (23), since we know that the local concentration gradient is unfavorable in terms of entropy, the noticeable contribution in Eq (23) should only in the surface layer. To study the effect of surface entropy, we go back to the formula of the entropy term and construct the free energy

$$F\{n\} = \int \{f^*(n(x)) - Tn(x) ln \frac{\hat{g}\psi}{\psi}\} d^3x \tag{50}$$

again we minimize it under the normalization condition

$$\int n(x) d^3x = N \tag{51}$$

By introducing the Lagrangian factor λ we get the minimization condition

$$\delta \left(F\{n\} - \lambda \int n(x) d^3x \right) = 0 \tag{52}$$

By variation, we obtain

$$\mu^* (n(x)) - T ln \left(\hat{g}\psi / \psi(x) \right) - \lambda = 0 \tag{53}$$

Remember in (20) we have defined $\Lambda = exp(-\lambda/T)$, then the Eq (53) becomes

$$exp \left(-\mu^* [n(x)]/T \right) \hat{g}\psi = \Lambda \psi(x) \tag{54}$$

We introduce another parametrically defined function $\nu(\psi)$ as $\nu \equiv exp\left(\mu(n)/2T\right)$ we can easily find the expression of free energy and $p^*(n, T)$, say

$$\begin{aligned} F &= \int \{f^* \left(n(x)\right) - n(x)\mu^* \left(n(x)\right) + \lambda n(x)\} d^3x \\ &= -TNln\Lambda - \int p^* \left(n(x)\right) d^3x \end{aligned} \tag{55}$$

$$p^*(n) = T\psi\nu(\psi) - 2T \int_0^\psi \nu(\psi) d\psi \tag{56}$$

We should note that the volume approximated Eq. (40) is a simplification of above general formula. Because the pressure $p^* \left(n(x)\right)$ equals zero outside the globule as well as inside its nucleus, that is , the value of $p^* \left(n(x)\right)$ differs from zero only within the surface layer of the globule, the second term on the right-hand side of Eq. (55) is proportional to the surface area. On the other hand, as $-T \ln \Lambda$ in the first term can be shown, just equals to $\mu^*(n_0)$, thus the free energy (55) can be rewritten as

$$F = N\mu^*(n_0, T) + 4\pi R^2 \sigma \tag{57}$$

and the surface term is derived as

$$\sigma = -\int_{-\infty}^{+\infty} p^* \left(n(z)\right) dz \tag{58}$$

where z is the coordinate normal to the surface. By using a very rough calculation, we can estimate the surface tension as

$$\sigma \simeq aB^2T/C^{3/2} \tag{59}$$

and the total free energy as

$$F \simeq -NTB^2/4C + N^{2/3}Ta(-B)^{4/3}/C^{5/6} \tag{60}$$

note the first term is of volume contribution and the second is the surface entropy one.

Now we can take some look on the effect of the surface term. Generally speaking, a globule-coil transition point lies in the θ-region within a interval of order $N^{-1/2}$ below the θ-point, and it is determined by the balance between entropy gain caused by chain extension and loss in the energy of volume link attraction. Near the critical region, the entropic (surface) effect is, of course, small, but it becomes essential near the θ-point as the energy contribution is also small. Indeed, the total free energy of the globule we obtained above can be rewritten according to the *viral* coefficients in Eq. (45)

$$F = F_{vol} + F_{surf} = -N\theta\tau^2(b^2/4C)[1 - |\tau_{tr}/\tau|^{2/3}] \tag{61}$$

where

$$\tau_{tr} \equiv (T_{tr} - \theta)/\theta \simeq -2.7a^{3/2}C^{1/4}/(bN^{1/2}) \tag{62}$$

is the deviation of the globule-coil transition point from the original θ point. Since it is proportional to $N^{-1/2}$, which is of the same order of magnitude of θ, we can say the transition point lies in a θ region. As $N \to \infty$, $T_{tr} \to \theta$ as expected. In practice, a typical deviation of the temperature T_{tr} from θ is of order 10K.

A key role in the coil-globule transition is played by the fact that the number of links N in a real macromolecule, even though large, is far less than infinity in a common sense compared to a typical thermodynamic system. As a result, the transition is continuous although a more or less sharp one with the width ΔT, which is characterized $F \sim T << NT$ near the critical temperature. From the expression for the free energy (61) we got, it is easy to derive

$$\Delta T \sim (\theta - T_{tr})C^{1/2}/a^3 \sim \theta C^{3/4}/(a^{3/2}vN1/2) \tag{63}$$

It can be seen that $\Delta T \to 0$ as $N \to \infty$, this validifies our treatment this transition as a phase transition.

Now we discuss the effect of another very important parameter, namely v/a^3, on the character of a globule-coil transition. As shown before, the v appears in the *virial* expansion formula in (39), being viewed as excluded volume of monomer, and a is nothing but the mean square spatial distance between the adjacent monomers. v/a^3 specifies the stiffness of the polymer chain: $v/a^3 \sim 1$ corresponds to $l \sim d$, i.e., the thickness of the monomers is of the same order of adjacent distance; $v/a^3 << 1$ corresponds to $l >> d$. The former and later ones are often called flexible chain and stiff chain due to their appearance. Back to the *virial* coefficient B and C, we have already obtained

$$B \sim v\tau, \quad C \sim v^2, \quad \tau = (t - \theta)/\theta \tag{64}$$

Accordingly, Eq (62) for the transition point and Eq (63) for the transition interval width may be rewritten in the form

$$\tau_{tr} \sim -(a^3/Nv)^{1/2}, \quad \Delta T \sim \theta(v/Na^3)^{1/2} \sim \theta|\tau_{tr}|(v/a^3) \tag{65}$$

we can see the situation essentially depends on the characteristic dimensionless parameter v/a^3, when $v/a^3 << 1$(i.e., for stiff chains), $\Delta T/\theta << |\tau_{tr}|$. This means the globule-coil transition in a stiff polymer chain proceeds in a relatively narrow temperature interval, clearly separated from the θ-point, on the other hand, a flexible chain gives $\Delta T/\theta \sim |\tau_{tr}|$ because $v/a^3 \sim 1$, the globule-coil transition is relatively smooth and the transition interval includes the θ-point. Both situations are shown in Figure (3).

This completes our discuss on the theory of globule-coil transition of the macromolecules. Before ending, we would like to make a few remarks about natural globular molecules.

The most widespread example of a biological globules are the globular proteins-enzymes, immunoglobulines, etc. The dense conformation in these systems is due to the volume interactions of the links surrounded by the solvent. Another interesting example of a biological globule is the double-helix molecule of DNA and RNA located in the head of the phage. In this system the volume interaction is basically repulsion.

The main feature of biopolymers is the heterogeneity of their primary structure. Protein molecules have unique primary structures and show extremely high specificity in their biochemical activity. However it is still not very clear whether the thermodynamic properties of proteins under arbitrary conditions are sensitive to the details of the primary structure. (in the case of DNA the primary structure, apparently, does not play any role in the spatial structure, because the complementary pairs are situated in the interior of the double helix).

In conclusion, this article introduced a model theory based mainly on the work of I. M. Lifshitz *et al*, such a theory describes with qualitative accuracy the homo-polymer chain in fairly simple situations and can, apparently, be used as a basis for further, more realistic approximations. As to biopolymers, this theory can stimulate the formation of interesting problems.

References

[1] I. M. Lifshitz: *Some problems of the statistical physics of polymer chains with volume interaction*, Review of Modern Physics, Vol. 50, No. 3, July 1978

[2] P. J. Flory: *Principles of polymer chemistry*, Cornell University Press, 1953

[3] M. Plischke, B. Bergersen: *Equilibrium statistical physics*, 2nd Edition, World Scientific Publishing Co. Pte. Ltd., 1994

[4] A. Yu. Grosberg, A. R. Khokhlov: *Statistical physics of macromolecules*, AIP press, New York, 1994

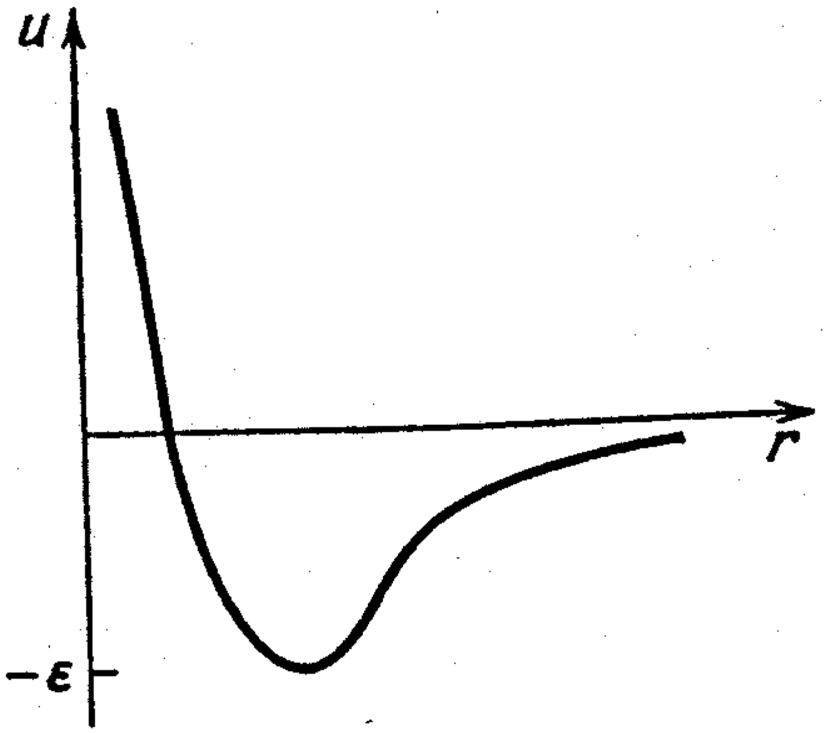

Figure 1: Typical potential of interaction between links in a standard polymer chain model.

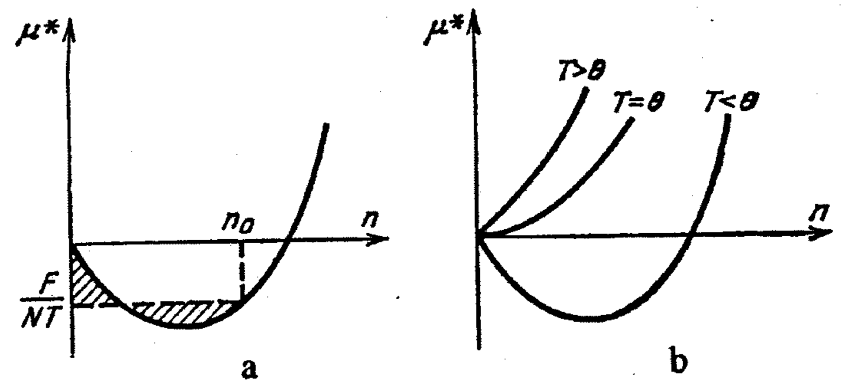

Figure 2: Interpretation of the conditions defining the density (43) and (41) of a large globule. (a) Equality of shaded areas. (b) Variation of $\mu^*(n)$ plot with temperature.

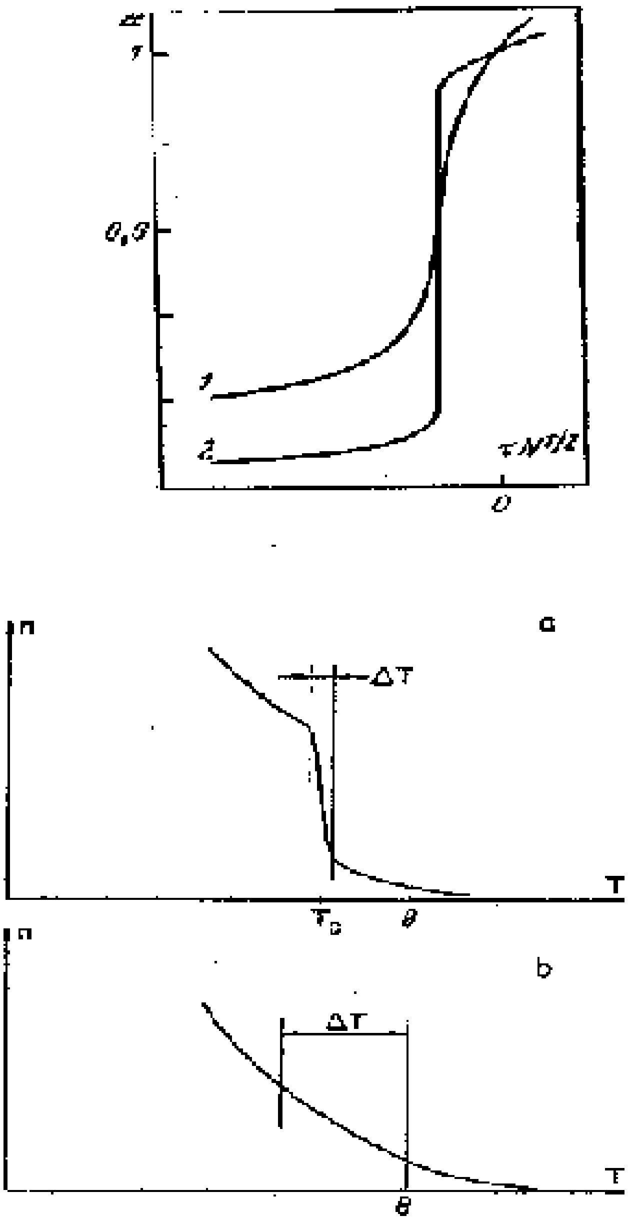

Figure 3: Upper one shows the dependence of the swelling parameter of a macromolecule in the region of the coil-globule transition for very flexible chains, $v/a^3 = 0.2(1)$, and very stiff chains, $v/a^3 = 0.01(2)$. The lower two graphs show the temperature dependence of average density in the coil-globule transition region (a) stiff $v/a^3 << 1$ (b) flexible $v/a^3 \sim 1$.