Spin Glasses

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Abstract

The understanding of random magnetic systems known as spin glass has been one of the major challenges in condensed matter physics over that past two decades. Study of this state can be thought of as a first step towards understanding more complex and disordered systems.This paper reviews some of the theoretical developmets in this field and tries to answer the question how well are these systems understood.Some of the experi mental properties and a theoretical definition of these system are introduced followed by a review of theoretical models.Emphasis is on meanfield theories and the statistical formulations needed to understand these systems.Some dynamical models and the applications of RG group and simulation methods follow this.The paper ends with the application of SG models to related systems in Physics and other fields especially biological systems.

1 INTRODUCTION

Understanding the random magnetic systems known as Spin glasses has been one of the major challenges in Physics over the last two decades.How much is it under stood is still a debated question.The major theoretical interest in this field was started with the observation of the a cusp in a.c susceptibility suggesting some kind of phase transition.The questions which required answers were (1)What is the order parameter? (2)What is the broken symmetry? (3)How we have to modify statistical mechanics to deal with a system containing disorder? (4)How to take into consideration the broken ergodicity exhibited by these systems ? In this paper we try to answer some of these questions. [1]

The organisation of this paper is as follows.The first part of the paper deals with a concrete definition of the spin glass This is followed by a review of the possible exchange mechanisms that are present. The next section is about the experimental properties of the spin glasse.The properties discussed are mag netic susceptibility, specific heat (all macroscopic pr operties measured at large time scales) , neutron scattering, Mossbauer effect, (microscopic properties measured on short time scales).This is followed by the theoretical models used to understand the behaviour of this magnetic state. The next section deals with the application of the spin glass models to related fields in physics and in other subjects.

2 What are spin glasses:An "accurate" definition

The most apropriate working definition for spin glasses is "Spin glasses are are random magnetic systems charaterized by a random freezing of spin at a well defined temperature T_f" . ,and this definition is incomplete without understanding the charateristics of t he system near T_f.There are two ways of creating random magnetic systems.First kind are crystaline in which the magnetic impurities(In spin glasses the concentration of the magnetic ions has to be small) substitute the nonmagnetic element at random positions.The second method is by crystallographically disordering(making it amorphous) an intermetalic compound. This again is the cause of another important charateristic of the spin glass systems namely the presence of "frustration". A frustrated system is one which, not being able to achieve a state that satisfies entirely its microscopic constraints(or topologies),posses a multiply equally unsatisfied states. For example consider the square lattice shown in the figure.Here + corresponds to J >0(ferro) and - to J <0 (anti ferro).The individual bond energies are minimised if the two spins connected by any arbit rary bond $< ij >$ are parallel to each other for J >0 and antiparallel for J <0.Notice that all the bond energies in the right figure can be minimized simultaniously whereas the same is not possible for the left figure.Therefore theright spin configuration is unfrustrated whereas the left configuration is frustrated. A frustrated system therefore has no unique microscopic arrangement for its ground state; there is an essentially infinite number of equivalant states that can be adopted. Some consequences of the presence of frustration in a magnetic ststem is metastability ,hysteresis ,time dependent relaxation towards equilibrium state, or dependence on samples thermal or magnetic histories.

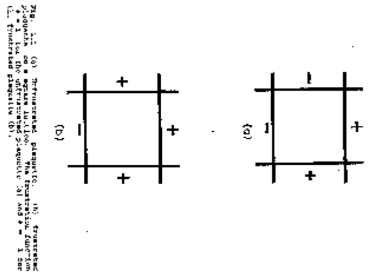

Figure 1: frustration,First fig unfrustrated and the second frustrated

The last term that need to be explained is spin freezing near T_f.This means below T_f the spins are no longer able to fluctuate rapidly with time but instead are locked into random orientations.To put random spin freezing mathematically

$$< S_i >_{\tau_0} = {\tau_0}^{-1} \int_0^{\tau_0} S_i(t)dt \tag{1}$$

are nonzero for some range ofτ_0.Their mean square

$$q(\tau_0) = N^{-1} \sum_i < S_i >^2_{\tau_0} \tag{2}$$

is thus nonzero for some τ_0.The absence of long range order implies that

$$\begin{aligned} \sum_i g_i < S_i >_{\tau_0} &= 0 \\ g_{ij} < S_i S_j > &\rightarrow 0\, as |i-j| \rightarrow \infty. \end{aligned} \tag{3}$$

for all simple periodic functions g_i and g_{ij}

2.1 Possible magnetic interactions that are present in the Spin glasses

In spin glasses due to the low concentration of the magnetic impurities the presence of direct exchange interaction(due to the overlap of the wave functions of adjascent magnetic impurities) can be neglected. In metallic spin glasses in teraction is predominantly due to the RKKY interaction.In RKKY interaction the magnetic ion induces an oscillatory spin polarization in the conduction electr ons in its neighbourhood.The exchange iteraction has the form

$$J(R) = \frac{J_0 cos(2k_F R + \phi_0)}{(k_F R)^3}, asR \rightarrow \infty \tag{4}$$

Where J_0 and ϕ_0 are constants and k_F is the Fermi wave number of the host metal.Note that since the distance between the spins are random , some of the interactions of a considered spin withe other spins will be positive , favouring parallel alignment , some negative favouring antiparallel alignment; thus no spin alignment can be found that is satisfactory to all exchange bonds. So we see that the frustration is a natural consequence of the presence of RKKY interaction and randomness. The other possible exchange interaction that can be present is the Superexchange which is mediated by a ligand.(The overlap between the orbitals of the metal atom with that of the non meatlic element that is present is reponsible for this).This is the major mechanism present in insulating spin glasses.

3 Experimental properties of spin glasses

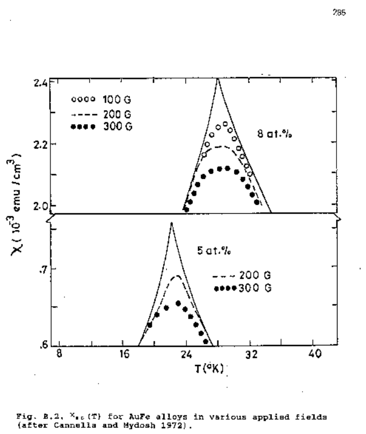

Figure 2: a.c susceptibility, Taken from Mydosh

The figure shows the low field a.c susceptibilty data.The figure has two properties which are charateristics of spin glasses.They are the sharp cusp at a temeprature T_f and the fact that the cusp is extremely magnetic field dep endent and the cusp gets flattened even at very low fields.The temperature $T_f$corresponding to the cusp of the a.c susceptibility is found to be an increasing function of concentration of the magnetic constituent in the alloy. It should be noted that for spin glasses the d.c magnetisation is history dependent and there is always a relaxation in the case of field cooled state to zero field cooled state and there are numerous theories and empirical relations for relaxation process and the corresponding relaxation time.In all cases it is found that the relaxation process is a slow one and availability of equlibrium states in this case is high ly doubtful. Another important imformation that we get from the a.c susceptibilty measurement is the frequency dependence of T_f.It is found that T_f increases with ω.It has been shown that The presence of large relaxation times of these samples was one of the first in dicators that for these materials the coarse-grained free energy in the space of apropriate phase space coordinates exhibits many "valleys".

The assumed freenergy valley is as shown(Mean field theories also predict this but in this case in thermodynamic limit the height of the valley diverges.But for real systems it is finite) Eventhough the presence of the sharp cusp in the a.c

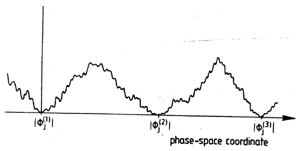

FIG. 4. Schematic plot of the coarse-grained free energy of a spin glass plotted vs a phase-space coordinate which measures the projection of the considered state on a particular ordered state. From Binder (1980a).

Figure 3: Free energy valley Taken from Binder and Young

susceptibility prompted the experimentalists to propose the existance of a phase transition at T_f the specific heat measurements put some doubts into the existane of the phase transition.The spe cific heat of spin glasses exhibit a broad peak at temperatures exceeding the freezing temperature by about 20%. This was an indicator that if there is a phase transition at T_f it is not of a con ventional type and the critical behaviour is subtle. It is interesting to note the the resistivity data also shows a broad peak at a temperature greater than T_f. The Mössbaur spectrum of the spin glass materials were probably the first demonstration of the existance of a phase transition in these systems.In this case the broad spectrum observed above T_f splits into a "six-fingured" patte rn at a sharply defined T_f.The intensity distribution of the Mössbaur spectrum gives a random distribution of the magnetic moments confirming the fact that in spin glasses below T_f the spin freeze in random directions. transition).

Neutron scattering suggests the presense of hysteresis and magnetic remanence(the history dependence of the magnetic properties) .The remanent magnetisation is found to decay very slowly.These two give information regarding the irreversible behaviour in the temperature region of the freezing transition and at lower temperatures.

These were some of the important experimental facts about the spin glasses which gave some idea to the theoretitians the kind of results that they should expect.

4 Spin glass theory

This section attempts to review some of the successful models used to explain the spin glass properties.The section is divided into two parts.The first part deals with the basic theoretical methods required for all study of diordered systems like spin glasses.The second section deals with the specific models for spin glasses.The next section deals with the solution of these models.The major emphasis is on meanfield theories. last section deals with the results of some computer simulations,renormalisation group applications .

4.1 General theoretical concepts

4.1.1 Averaging in random systems

One of the major difficulty in dealing with the spin glass state is that we are dealing with a random system and we should average over the disorder. Let us suppose that we have a system described by some statistical variables S_i(i=1,2...N).Since the system is random we need to specify the variables{x} de scribing randomness.So when we are taking the statistical averaging kind of averaging to be done depends on the time of observation.If $\tau_{exp} >> \tau_{dis}$

(τ_{dis} is the fluctuation time for random variables) then the free energy of the system becomes

$$F = -k_B T ln[Z\{x\}]_{av}$$
$$Z\{x\} = Tr_{S_i} exp[-\mathcal{H}\{x, S_i\}/k_B T] \quad (5)$$

This is called an *annealed* average.In the spin glass case usually $\tau_{dis} >> \tau_{exp}$ and in this case the random variables take unique values as the statistical variables fluctuate.In this case one evaluates the partition function for a particular configuration of the random variable Z{x}.This limit is known as the *quenched average*.Since in this case it is impossibly difficult to keep track of a large number of {x} we replace Z{x} or M{x} (M is the magnetisation) by some kind of annealed average. (self averaging),it is worth mentioning that in the most sucessful mean field theory this self averaging is violated by some quantities.So in this case

$$[M]_{avg} = [< S_i >_T]_{avg} \quad (6)$$

here $< .. >_T$denotes the statistical average and $[..]_{avg}$ denotes the configuration average. Another thing to be noted is that we need the average free energy that is $-k_B T ln[Z\{x\}]_{avg}$(note that the average cannot be performed directly since the random variable is inside the logarithm).This kind of averaging is difficult to do and a trick known as the *replica trick* is used. This replica aproach rests on the use of the exact relation

$$[lnZ\{x\}]_{avg} = \lim_{n \to 0} \frac{1}{n}([Z^n]\{x\}]_{avg} - 1) = \lim_{n \to 0} \frac{\partial}{\partial n}[Z^n\{x\}]_{avg} \quad (7)$$

for positive integer n one can express $Z^n\{x\}$ in terms of n identical replicas of the system

$$Z^n\{x\} = \prod_{\alpha=1}^{n} Z_\alpha\{x\} = \prod_{\alpha=1}^{n} exp[-\mathcal{H}\{x, S_i^\alpha\}/k_BT]$$
$$= exp[-\sum_{\alpha=1}^{n} \mathcal{H}\{x, S_i^\alpha\}/k_BT] \quad (8)$$

where Z_α is the partition function of the αth replica.For positive integer n it is simple to carry out the average $[..]_{avg}$ and on e can expres s the result formally in terms of an effective hamiltonian $\mathcal{H}_{eff}(n)$ that nolonger contains disorder and is translationally invariant,

$$Z_n = [Z^n\{x\}]_{avg} = \underset{\{s_i^\alpha\}}{Tr} \quad exp[-\mathcal{H}_{eff}(n)/k_BT] \quad (9)$$

For example for a random bond problem(one in which the exchange constants J_{ij} are chosen according to a fixed probability distribution $P(J_{ij})$ for zero magnetic field Ising model

$$\mathcal{H} = \frac{-1}{2}\sum_{ij} J_{ij} \ \mathbf{S_i}.\mathbf{S_j} \quad (10)$$

in this case the partition function is

$$Z_n = \underset{\{s_i^\alpha\}}{Tr} \prod_{<i,j>} dJ_{ij}P(J_{ij})exp\left[\frac{J_{ij}}{k_BT}\sum_{\alpha=1}^{n} S_i^\alpha S_j^\alpha\right] \quad (11)$$

From a Taylor expansion of this we can get

$$\mathcal{H}_{eff}(n)/k_BT = -\frac{1}{2}\sum_{ij}\sum_{k=1}^{\infty}\frac{1}{k!}\frac{J_{ij}^{cum}(k)}{(k_BT)^k}\left[\sum_{\alpha=1}^{n} S_i^\alpha S_j^\alpha\right]^k \quad (12)$$

Where $J_{ij}^{cum}(k)$ is the kth cumulant of J_{ij}

$$J_{ij}^{cum}(1) = [J_{ij}]_{avg} = \bar{J} \quad (13)$$
$$J_{ij}^{cum}(2) = [J_{ij}^2]_{avg} - [J_{ij}]_{avg}^2 \equiv (\Delta J_{ij})^2 \quad (14)$$

Now since $\mathcal{H}_{eff}(n)$ is an effective Hamiltonian of a translationally invariant problem without disorder and all methods of statistical mechanics for ideal pure systems can now be applied. In the mean field aproximation one replaces the terms $S_i^\alpha S_j^\beta$ by $S_i^\alpha < S_j^\beta >$,$S_i^\alpha S_i^\beta S_j^\alpha S_j^\beta$ by $S_i^\alpha S_i^\beta < S_j^\alpha S_j^\beta >$, $S_i^\alpha S_i^\beta S_i^\gamma S_j^\alpha S_j^\beta S_j^\gamma$by $S_i^\alpha S_i^\beta S_i^\gamma < S_j^\alpha S_j^\beta S_j^\gamma >$ and soon.So the order parameters

$$m_\alpha =< S_j^\alpha >, q_{\alpha\beta} =< S_i^\alpha S_j^\beta >_{\alpha\neq\beta}, q_{\alpha\beta\gamma} =< S_j^\alpha S_j^\beta S_j^\gamma >_{\alpha\neq\beta\neq\gamma} \quad (15)$$

another important fact to consider is that we need the analytic continuation of$\mathcal{H}_{eff}(n)$ to real positive n near n=0 is nonunique.Note that eventhough $\mathcal{H}_{eff}(n)$ is invariant under permutations of the indices of the replicas,as long as n is a positive integer,it is not obvious that the symmetry is preserved under the limit $n \rightarrow 0$.This breaking of symmetry is known as *replica symmetry breaking*.

4.1.2 Ergodicity breaking

[2] For spin glasses ergodicity is broken.Macroscopic irreversibilty evident as hysteresis, remanence and history dependence points to ergodicity breaking.And it should be noted that the different states(macroscopic) or phase space regions are not related by a symmetry of the Hamiltonian.They appear because of the *accidental degeneracy* arising out of the presence of randomness and frustration.So any complete theory of spin glasses should take this ergodicity breaking into consideration.

4.2 Theoretical models

In this section different theoretical models used in spin glass theory is discu ssed and their attempted solutions are given.No detailed mathematical steps are given.Only the concept involved in the model, assumptions and results are discussed. first of these kind of models is the Edwards-Anderson model. Here a classical ising hamiltonian is used and mean field theory is used to determine the order parameter defined by

$$q_{EA}(t) = [< S_i(t)S_i(0) >]_{avg} \tag{16}$$

.They replaced $q_{EA}(t)$by its long time limit $q_{EA} = [< S_i >^2]_{avg}$ and determined q and other thermodynamic functions using a mean field theory.Eventhough they could predict the cusp in susceptibilty quantitative agreement with experiments were far from being corrct.This was atributed to the inadequacy of the mean field theory that they used rarther than to the model.First modification to the Edwards-Anderson model was done by Sherrington and Kirkpatrik.They used the infinite range(mean field theory is exact in this limit) Edwards-Anderson model and used the replica trick as discussed before and assumed the replica symmetry. Here the ising model hamiltonian is used with a gaussian probability distribution for the exchange constant given by

$$P(J_{ij}) = [(2\pi)^(1/2)J]^{-1}exp[_(J_{ij} - J_0)^2/2J^2] \tag{17}$$

Since the model is infinite range to ensure extensive nature of free energy $J_0 = \tilde{J}/N$ $J = \tilde{J}/N^{1/2}$ The basic procedure of the metod is as follows. the free energy averaged over all cofiguarations is given by

$$[F]_{avg} = -k_B T \lim_{N \to \infty} \lim_{n \to 0} (1/n)\{[z^n]_{avg} - 1\} \tag{18}$$

where

$$[z^n](\{J_{ij}\})]_{avg} = \int \prod dJ_{ij} P(\{J_{ij}\}) z^n(\{J_{ij}\}) \tag{19}$$

The partition function

$$z^n(\{J_{ij}\}) = Tr_{\{s^\alpha}\} exp[-(1/k_B T) \sum \mathcal{H}^\alpha] \tag{20}$$

corresponds to the replica hamiltonian

$$\mathcal{H}^{\alpha} = -\sum J_{ij} s_i^{\alpha} s_j^{\alpha} - H \sum s_i^{\alpha} \tag{21}$$

where α(α=1,2..n) labels the replicas and i,j(i,j=1,2..N) denotes the spins.Using the gaussian probability distribution for J_{ij} and integrating over J_{ij} and using the Hubbard-Stratonovich transfomation and assuming that in the lim n$\rightarrow$ 0 the limits N$\rightarrow$ ∞ and$n$ $\rightarrow$0 can be interchanged and doing the integrals resulting out of Hubbard-Stratanovich transformation using steepest descent method we get

$$(F/k_BTN) = -(\tilde{J}^2(1-q^2)/4k_B^2T^2) + (\tilde{J}_0 M^2)/(2k_BT) - 1/(\sqrt{2\pi})$$
$$\times \int dzexp(-z^2/2) ln2cosh[(\tilde{J}_0 M/k_BT + \tilde{J}z \sum q^(1/2)/k_BT + H/k_BT] \tag{22}$$

where

$$M = \lim_{n\rightarrow 0} M_{\alpha} = (2\pi)^{-1/2} \int dzexp(-z^2/2)$$
$$\times tanh[\{\tilde{J}_0 M/k_BT\} + \{\tilde{J} q^{1/2} z/k_BT\} + H/k_BT] \tag{23}$$

and

$$q = \lim_{n\rightarrow 0} q_{\alpha\beta} = (2\pi)^{-1/2} \int dzexp(z^2/2)$$
$$\times tanh^2[\{\tilde{J}_0 M/k_BT\} + \{\tilde{J} q^{1/2} z/k_BT\} + H/k_BT] \tag{24}$$

In the pase diagram(Whose axes are $\tilde{J}_0/\tilde{J}$ and $kT/\tilde{J}$)three phases appear. 1.Paramagnetic phase charaterised by M=0 and q=0
2.The SG phase(M=0 and q$\neq$0)
3.The ferromagnetic phase M$\neq$0 and q$\neq$0.
The SK solution has a specific heat cusp at T_f which is in disagreement with the experimental data and the SK solution has negative entropy at low temperatures which is not physically acceptable.This is atributed to the breaking of the permutation symmetry between the replicas.Note that in the SK solution we asumed $q_{\alpha\beta} = q \; \forall \; \alpha, \beta$and this is true only if the permutation between the replicas which is true when n is a positive integer is valid in the n$\rightarrow$ 0 limit.

To get a correct solution of the SK model Parisi used a replica symmetry breaking scheme.(This essentially means $q_{\alpha\beta} \neq q \; \forall \; \alpha, \beta$).First one begins with the SK replica symmetric matrix.After the first stage the matrix q consists of $n/m_1/timesn/m_1$ blocks each of size $m_1/times\, m_1$.At the next stage,the off diagonal blocks q_0 are left unmodified where as the diagonal blocks q_1 are further subdivided as q_2 continuing the above procedure we subdivide the diagonal blocks q_2 further into smaller blocks and so on.The process is terminated after R such iterations,with the smallest diagonal block q_R of size $m_R \times m_R$.In the limit n$\rightarrow$ 0 .This scheme along with the physical requirements

$$\lim_{n\rightarrow 0} (1/n)[\sum q_{\alpha\beta}^2] < \infty \tag{25}$$

so that the configuration averaged free-energy is finite

$$\sum q_{\alpha\beta} = \sum q_{\gamma\beta},\ \alpha \neq \gamma \tag{26}$$

so that the free energy is an extremum with respect to q.

$$\begin{array}{l} -lim\ \ (1/n)\sum q^2_{\alpha\beta} \geq 0 \\ n \to 0 \end{array} \tag{27}$$

so that at high temperatures the minimum of the free energy is located at $q_{\alpha\beta} = 0$.In this case since $q_{\alpha\beta}$=q(m_i) and in the limit n$\rightarrow$ 0 m_i is a continuous(x) variable and the order parametr q(x) is a function of x.The free energy can be found from the solution of the differential equation

$$\frac{\partial F}{\partial x} \ = \ -\frac{1}{2}\frac{dq}{dx}\ [\frac{\partial^2 F}{\partial H^2} + x\ \left(\frac{\partial F}{\partial H}\right)^2] \tag{28}$$

where F(x,H) is a generaliaed free energy such that F(0,H) gives the actual free energy.The corresponding boundary condition is

$$F(1,H) = ln[2\,cosh(\beta H)] \tag{29}$$

The predictions of the Parisi calculation of low temperature behaviour of q,entropy,susceptibility are in good agreement with numerical simulations.Parisi's aproch of replica symmetry breaking also led to the inclusion of an interesting mathematical concept to physics known as the ultrametricity.This can be understood in the following way.As discussed before the spin glass phase is charaterised by the existance of a large number(infinite when $N \rightarrow \infty$)number of equilibrium states α=1,2..almost degenerate(free energy valleys seperated by free energy barriers becoming infinity in the thermodynamic limit for the mean field theory)This essentially means the ergodicity is broken.If we define the overlap between the two states as

$$q_{\alpha\beta} = \frac{1}{N}\sum m_i^\alpha m_i^\beta\ \ where\ m_i^\alpha =< S_i >_\alpha \tag{30}$$

.Ultrametricity implies if we have three states α, β, γ and we calculate the overlap between them namely $q_{\alpha\beta}$, $q_{\beta\gamma}$, $q_{\gamma,\alpha}$ then in Parisi's scheme it turns out t hat they cannot all be different; either all three are equal or the two smaller ones are the same.This is the property of a matric space(known as the ultrametric space) studied in detail in mathematics.It is also noted that the quantities like parisi's order paremeter q(x) looses it's self averaging nature.(depends on the particular configuration of the random variables x which charaterize the system). The enrgy levels in this scheme has got a tree like structure.

Another way of solving the SK model is without using the repllica trick.This known as TAP solution uses the local magnetisation.This ueses a mean field theoretic derivation analogous to Weiss molecular theory but in this case the fluctuations

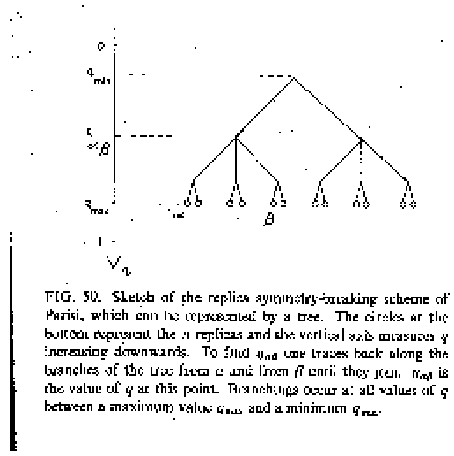

Figure 4: Tree like structure of energy in Paisi scheme, Taken from Binder and Young

cannot be neglected.The formalism uses the expansion of free energy in a series of diagrams.Here a simplified and physically more understandable treatment is given.Another way of deriving the TAP equations is using the free energy expansion in pairs and loops.The pairs contribute terms of ordrer N and loops N/z.And neglect all the loop terms.In the infinite range limit(N$\rightarrow \infty$) this is exact.Here mean fild theoretic derivation is given.

The hamiltonian can be divided into

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{fluc} \tag{31}$$

where

$$\mathcal{H}_0 = \sum J_{ij}(M_iM_j - M_iS_j - M_jM_i) \tag{32}$$

is a soluble mean-field hamiltonian and

$$\mathcal{H}_{fluc} = -\sum J_{ij}(S_i - M_i)(S_j - M_j) \tag{33}$$

Eventhough the conventional mean free theory drops the fluctuation term TAP doesnot drop this term and in the z(coordination number)$\rightarrow \infty$ we get the TAP equations(Obtained by minimising the free energy with respcet to M_i)

$$M_i = tanh(\beta h_i) \tag{34}$$

where

$$h_i = \sum J_{ij}M_j - \beta \sum J_{ij}^2(1 - M_j^2)M_i \tag{35}$$

. These equations can be interpreted in this way.The field produced at the j-th site by the i-th spin is $J_{ij}S_i$.This field induces a magnetisation $J_{ij}S_ix_j$ at the j-th site,the later magnetisation producing a field $J_{ij}^2S_i\chi_j$ and hence the latter field is called the reaction field.So the magnetisation equation implies that the magnetisation at an arbitrary site i is determined by the cavity field which is the field in the absence of the i-th spin.In the spin glass the reactio field is of the same order as the Weiss field.(Reaction field assuming $J_{ij} = \pm J$, $R \approx zJ^2\chi M$ near $T \sim T_f$, $\chi \approx 1/T_f$ and since $T_f \approx z^{1/2}J, R \approx z^{1/2}JM$ and the Weiss field $W \approx z^{1/2}JM$ for $T \sim T_f$)

The solution of the TAP equations is nontrivial since they are a set of N coupled equations which have to be solved self consistantly(This is done numerically).TAP gives stable(marginally stable) solution below T_f.The TAP equations have exponentially large number of solutions each corresponding to a local minimum of the free energy.

As mentioned before the spin glass state is not in actual equilibrium.So many dynamical theories were attempetd.The Sompolinsky's aproach is one of the successful ones.Here the soft spin version of the EA model is used , a phenomenological Langevian equation is used , the validity of the fluctuation-dissipation theorm is assumed and introduced two order parameters.In this aproach the replica method of getting the configuration average can be substituted by a functional integration aproach.The results are in good agreement with the mean field theory results.

In this section we discuss one of the most recent models in spin glass known as the "droplet model".This is one of the first short range models that has been solved analytically.In this model the spin glass is described by the exitations of its ground state known as the droplets.These are regions in the sample where spins are in the opposite direction as that of the ground state. They propose some scaling form for the free energy of these droplets in terms of their fractal dimension(droplets are fractals).It is interesting to note that the ground state in unique(rather doubly degenerate:states are related through global inversion of spin) in this model

These were the major models used in the spin glass theory.Some other models which are also used are discussed below.No attempt is made to give the results of these models.Random energy models is one such model which can be solved without using the replica trick.Here we assume the system has 2^N random (gaussian distribition) energy levels.This is an exactly solvable model and can be used to get some qualitative predictions about the spin glass ground s tate and can be used to check the validity of the replica aproach.(This model can be solved using both replica aproach and without using this).The other model of interest is the spherical model.Here we assume that the spins satisfy $\sum S_i^2$=N where N is the number of spins in the system.This model also can be solved without using the replica trick.(The drawback is the classical (not quantum) spherical model gives negative entropy at low temperature).The other interesting model that is cosidered is the vector SG model.Here the spins are treated as classical vectors in m dimensions.This gives an interesting prediction of both ferromagnetic and SG phase coexistance for a particular temperature range.

There have been attempts along the line of phenomenological Ginzburg-Landau-Wilson Hamiltonian.Most of these models are very difficult to solve and does not give much more information than the SK model ,in replica scheme.If we try to avoid the replica scheme to facilitate analysis we are forced to choose simpler hamiltonians and the results of these models cannot be trusted since it is doubtful whether these models contain all the ingradients of spin glass.

4.3 Renormalization group applications

In this section the Renormalisation group applications to the SG models has been described.Before getting into the details it is instructive to note that the power of renormalization group lies in its ability to predict the critical exponents accurately.(At least for ordered systems).And the application of reno rmalization group requires some kind of scaling(or atleast postulate one).In spin glasses it is very doubtful whether any of these conditions are satisfied.So most of the time renormalzation group methods use heuristic scaling hypothesis(in real space renormalization case) or heuristic phenomenological ginzburg-landau-wilson hamiltonians in the case of field theoretic renormalization case.And the uncertainity regrarding the values of critical exponents as mentioned before and the heuristic assumptions that is made in applying RG tecniques makes its re sult not readily accepted unless it is substantiated by some sort of numerical simulation.Still present day research on SG uses RG aproaches and considerable progress(at least to many who apply this) is made in the applicability of RG techniques to SG problem.(It is interesting to note that one of the most successfulmodels(droplet model) used for short ranged spin glasses uses many RG arguments.

Field theoretic renormalisation techniques and the ϵ expansion with ϵ =6-d were applied to the SG problem and the outcome has not been very sucessful.In some cases this yields compex critical exponents and the presence of dangerously irrelevant variables complicates the analysis.The real space renormalization group arguments were more successful.In this scheme the basic quantity is the distribution of exchange interaction J.After application of the RG tranformation the mean K_0 and the variance K of this distribution gets modified and the flow in the parameter space converges into some fixed point.If K$\rightarrow \infty$ and K_0/K $\rightarrow$ 0 we get a SG fixed point.The another aproach is the Migdal-Kadanoff bond-moving method.RG calculations predict T_f=0 for ising SG in d=2.It should be noted that since there are no natural expansion parameters in RSRG the errors are often uncontrollable.The most successful RG application has been the Motecarlo renormalization group calculation.

4.4 Computer simulation and numerical methods

As mentioned before most of the experiments done on SG to varify the theoretical predictions are computer experiments.It can be said without much doubt that for a SG theorist the most important experimental tool is a computer. The computer simulation or the so called "numerically exact" calculation is not beyond doubt.There were many instances in which two differnent aproaches leading to contradictory results.The fact that in computer we are studying a finite system to make predict properties of an infinite system makes it imperative to makesome aproximations regrading the extrapolation of the data.Scaling arguments used for this are treated with caution.The fact that even computer experiments has nothing to compare its predictions (for nonrandom systems there are good experimental data to compare), and the presence of a large number of ground levels and the long relaxation times which is a consequence of this makes the computation and analysis very difficult.Given all these draw back it must be accepted that computer is the most effective tool to understand the spin glasses.(This is evident in the pattern of the present day publication in this field most of these combine a RG scaling and simulation of short ranged models) ,although most of the people take the simulation results with caution.It is interesting to note that the relationship between computers and SG problem is symbiotic since better understanding of SG problem can be used to solve many optimization problems which are cruicial in the developement of fast and cheap computers.An example is given in the section on SG models to other systems.

In this paragraph a brief review of the simulations methods used is given.The importance lies in the fact that lower critical dimension and the understanding of the realistic models have been through computers.In computer simulation one begins with a well defined model and lets the system evolve following some prescription for time evolution.The montecarlo simulations using the metropolis algorithm is the most popular one in use.This consists of two steps.First select a spin configuration randomly.Then compute the energy change ,if any, by the flipping of the i th spin.If this is negative accept the solution or else reject it.In spin glass case this is complicated by the required averaging over the various configurations.The numerical analysis of the spinglass has been complicated by the presence of finite barriers in the free energy valley.So to get something sensible out of the simulations we have to take observation time to infinity and this requires a lot of computer time as well as memory.Simulation results have shown that T_f=0 when d=2 and is nonzero for d> 2.Simulation can also give values of critical exponents.Another method has been the "numerically exact" treatment using the transfer matrix technique to managably large but finite lattices on a computer.This calculates the partition function for a given finite lattice with a quenched random bond configuration using transfer matrix techniques.

4.5 Aplications of SG models

In this section a review of the applications of the spin glass model to other related systems in physics and other filds are discussed.The first example is the electric analog of spin glass known as the electric dipolar and quadrapolar glasses.So the theories developed for the spin glass can be used for the description of the electric moment in these systems.Another system that can be studied is the orientational order in the solid hydrogen.H_2 can be ortho and para depending on the total angular momentum.At room temperatures para is the dominant componant and a small fraction of ortho componant that is present in the solid hydrogen can cause orientational order and this can be studied using spin glass models.Another interesting application is in the superconducting Glass phase in Granular superconductors and Josephson junctions.Here superconducting grains are embeded in a host which can be a metal insulator or a superconductor.The superconducting grains interact via josephson coupling or by proximity effect.A suitable spin glass parameter in terms of phase is introduced and the SG like transition is charaterised by the random distribution of the frozen josephson current.

Travelling sales man problem is one of the classic examples of the application of the spin glass like models to non physics problems.The problem is for a sales man to visit given N cities and distance between any two cities,what is the shortest route that visits each city once and returns finally to the starting point.?This is one of the most important problems in combinatorial optimization and belongs to the class of so called NP complete problem. It is obvious that all the ingradients of spin glass problem are present in this problem.The energy valley of this (analogue of cost function) problem has similar structure as that of the spin glass.And the methods developed for the spin glasses especially the Random energy model has been used to get aproximates solutions to this problems in specialised cases.Spin glass model has been apllied successfully to other problems similar to TSP.

The other problems of interest that are modelled using SG models are prebiotic evolution and neural networks.In prebiotic evolution analogy is drawn between DNA formation from random polymers as a process involving randomness and frustration. In neural networks the Hopfields model uses a SG hamiltonian with state of the neuron as spin,the signal from i-th to j-th neuron(which is random) as J_{ij} and threshold as h_i.Spin glass solutions can be applied, though it is not clear ,to protein folding.

5 Conclusions

In this paper we have tried to understand something about the spin glass state and spin glass transition.Most of the formulation used were equilibrium mean field theories.The mean field theory solutions are well understood and the Parisi's solution seems to be exact.Clearly we understand this state qualitatively.Now theorists generally agree that the spin glass transition is indeed a phase transition with global inversion of symmetry is broken and there is ergodicity breaking associated with this.Numerical simulations and RG analysis also points to this and the lower critical dimension for ising model has been established as 2 and for the Heisenberg model as 3.(This i s disputed sometimes).The upper critical dimension for mean field theories is also established as 6.Another view that has strong presence is that spin glass phenomena is essentially a dynamic critical phenomina and more developements in that field is going to give us a better understanding of SG.The dynamical theories of short ranged models are hoped to reproduce the behaviour of real systems.Another interesting point to mention is the application of 1/f noise measurements done by Weissman[3] etc which confirmed the presence of Parisi like scheme of energy levels in spin glass.Present papers on this field concentrates more on the application of RG and simulation methods to Ising and non ising models and some papers are trying to take into consideration the quantum effects.(There is a tendency that whatever developements in Statistical physics happens peoples try to apply those to SG problem.)In this sense this state is not clearly understood and this is an open problem for anyone to try and solve.May be some phenominological model proposed might one day solve this problem completely.An optimist might say the present research on spin glass was successful in the sense that a qualitative understanding of this state has been achieved and this brought real innovative things like replica symmetry breaking and ultrametricity into physics and since there are more things to do we should continue the research.A pessimist also has room to continue research in this field in a way which can be summerised by a quote from Choudhary's book "although our brains have not been able to understand the SG very well we should not be disappointed;let spin glass be a tool for underst anding the brain".And the technological applications in computer design and neural networks are not to be overlooked.

References

[1] There are a lot of reviews on this subject.This paper mainly uses ideas from the ones given.No attempt is made to make the references comprehensive.More references can be obtained from the ones given below. .

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