Superconductor-Insulator Transition

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1 Introduction

In recent years, a lot of work has been concentrated on explorations of electronic systems. It is believed that understanding of the physics in 2D will give an insight into the problem of Hi-Tc superconductivity. The dimension d=2 turns out to be critical for both superconductivity and localization. Disorder-driven superconductor-insulator transition is governed by this two important effects.

The impossibility of long-range order in $d \leq 2$ was proven by Höhenberg[1] and Mermin and Wagner. Here is the general idea of a proof for Bose condensation. The static response function $\chi^s_{A,B}(k,k)$ satisfies the axioms of scalar product and an upper bound $\chi^s_{A,A^\dagger}(k) \leq (\beta/2)C_{A,A^\dagger}(k)$ where C is the equal-time correlation function.

Taking $A_k(t) = i(\partial/\partial\ t)\rho^\dagger_k(t)$ and $B_k(t) = \frac{1}{\sqrt{\Omega}}a_k(t)$ one can write the Schwartz inequality:

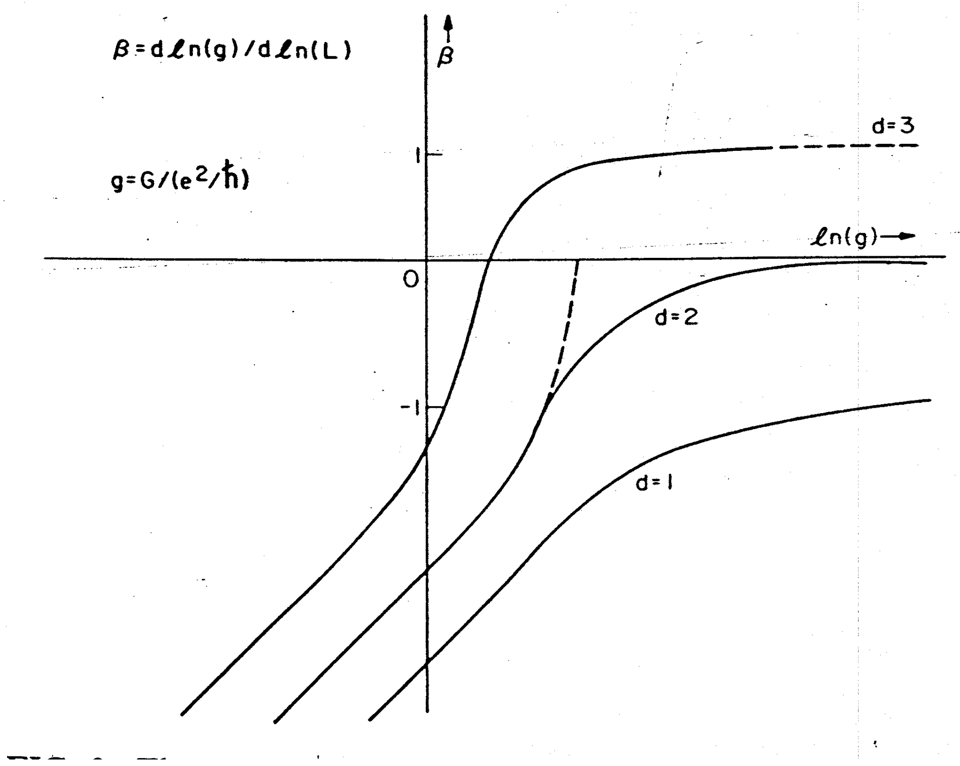
$$\frac{|\chi^s_{A,B}(-k,k)|^2}{\chi^s_{A^\dagger,A}(-k)} \leq \chi^s_{B,B^\dagger} \leq (\beta/2)C_{B,B^\dagger}(k) = 2n_k + 1.$$

$$\chi^s_{A,B}(-k,k) = -\sqrt{n_0}$$

Performing sum over all k-s in an infinite volume, due to the divergence in the integral for $d \leq 2$ the inequality can be satified only if $n_0 = 0$ or $T = 0$, i.e. no superfluid component is present. Similar, though more technical proof rules out the possibility of superconductivity.

However, this does not rule out the superconductivity completely: it can be restored below the Kosterlitz-Thouless transition temperature[3], when quasi-long-range order is established.

The idea of localization first appeared in a work by Landauer[2]. He considered a 1D system of obstacles, and found that the dimensionless resistivity $r = \frac{e^2}{\pi\hbar}R$ fails to obey the Ohm's law. Instead, one obtains $r = r_1 + r_2 + 2r_1r_2$ for the junction of two systems with resistivities r_1 and r_2 respectively. In the limit $r_1, r_2 \ll 1$ this reduces to Ohm's law $r \approx r_1 + r_2$, which implies that



the resistivity is proportional to the length of a sample. In the opposite limit $r_1, r_2 \gg 1$, the resistivity increases exponentially with the length of a sample.

It was conjectured[4] that in real metals, states can be either extended or localized with localiztion length ξ. It can be easily shown that in the limit of strong disorder, no extended state can exist.

The two limits can be interpreted as follows: Let L be the length of a sample and l — the electron mean free path.

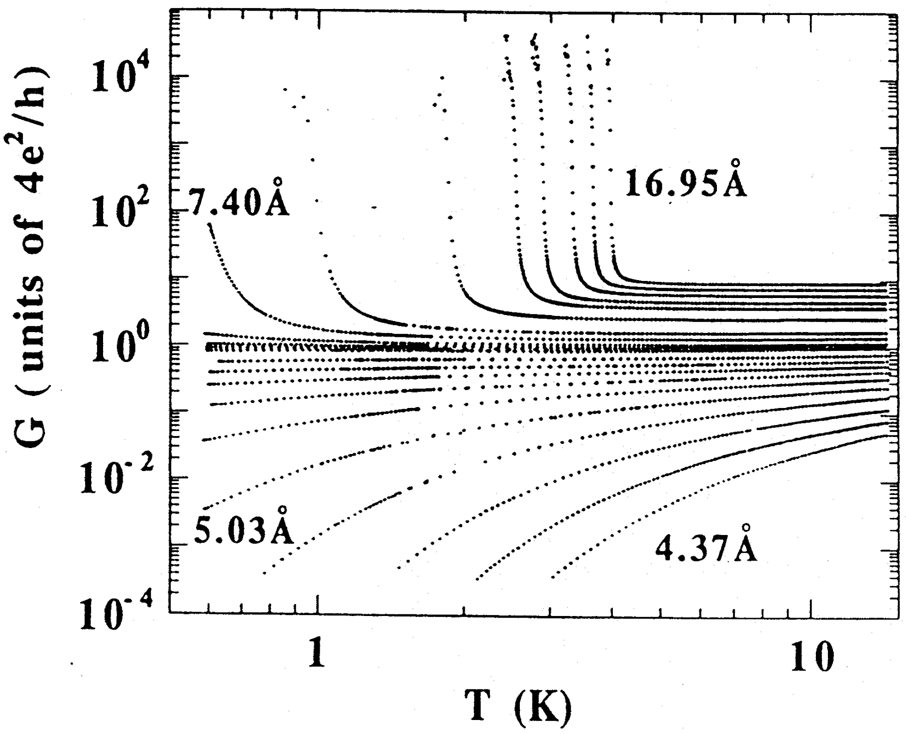
$$L \gg l : g(L) = \sigma L^{d-2}$$

$$l \gg L \gg \xi : g(L) \sim exp(-L/\xi)$$

We can draw a picture of the scaling function $\beta(g) = d \ln g / d \ln L$. We use the asymptotics above and interpolate in between.

The d=2 is a marginal case; because of the corrections to conductivity the $T = 0$ resistivity is infinite for arbitrarily weak disorder.

For SI-transitions the lower critical dimension is $d = 1 < 2$, and this problem does not arise



2 Experimental Evidence

The concept of superconductor-insulator transition has appeared from studies of interplay between superconductivity and localization in homogeneously disordered ultrathin films. It was observed that for some critical disorder/thickness system goes from superconducting state, characterized by zero resistivity at $T = 0$ and insulating state (σ=0 at $T = 0$). (Above is the plot of resistivity vs. temperature for varied film thickness)

The films on which the experiments were conducted can be divided into two classes: homogeneous and granular films (they are disordered on atomic and mesoscopic scales respectively). The resistivity curves for homogeneous films are monotonic in nature, while those for granular films demonstrate drops in resistivity in the vicinity of bulk transition temperature and "dips" - local minima in R(T) for some film thicknesses. The latter phenomenon is referred to as a "quasi-reentrant" superconductivity.

The transition can be tuned by disorder, carrier concentration, film thickness, or external magnetic field. In all cases, a transition between superconducting and insulating phases has been observed (It is sharp at T=0). This is an example of quantum phase transition (QPT).

Right at the transition in zero magnetic field there exists a limiting resistance close in its value to the quantum unit of resistance $R_Q = \hbar/(2e)^2$ (The exception is MoC systems, where $R_0 < R_Q$; in most other homogeneous films $R_0 \approx R_Q$. Besides, in many cases R_0 turned out to be experiment-dependent rather than material-dependent, which raises doubts about the reliablity of such results. Finally, what is measured in experiments is the resistance separatrix, which does not necessarily equal zero-temperature "marginal" resistance.).

Another experimental piece of evidence is duality. For points, equidistant from a transition on insulating and superconducting sides respectively one has:

$R_I/R_0 = R_0/R_S$ ($R_I(R_S)$ - resistance on the insulating (superconducting) side).

In the critical regime, resistance scales as:

$$R(\delta, T) = (\hbar/4e^2)Q[b\delta/T^{1/z\nu}],$$

where Q is the scaling function, δ is the parameter which tunes the transition, ν and z are the critical exponents. The exact values of z and ν vary from experiment to experiment.

The magnetic field scaling suggests that the resistance is a function of $|B - B_c|/T^{1/z\nu}$

The most extensive study has been performed on InO_x. The results obtained are in excellent agreement with the dirty boson model. R_0/R_Q was measured to be ≈ 1.1.[5,6] Both Kosterlitz-Thouless transition temperature T_c (the observed superconducting transition temperature) and the mean-field value of T_{c0} (from fitting conductance data to Aslamazov-Larkin theory) were measured, and it was found that at the transition, where T_c goes to 0, T_{c0} remains finite.[7]

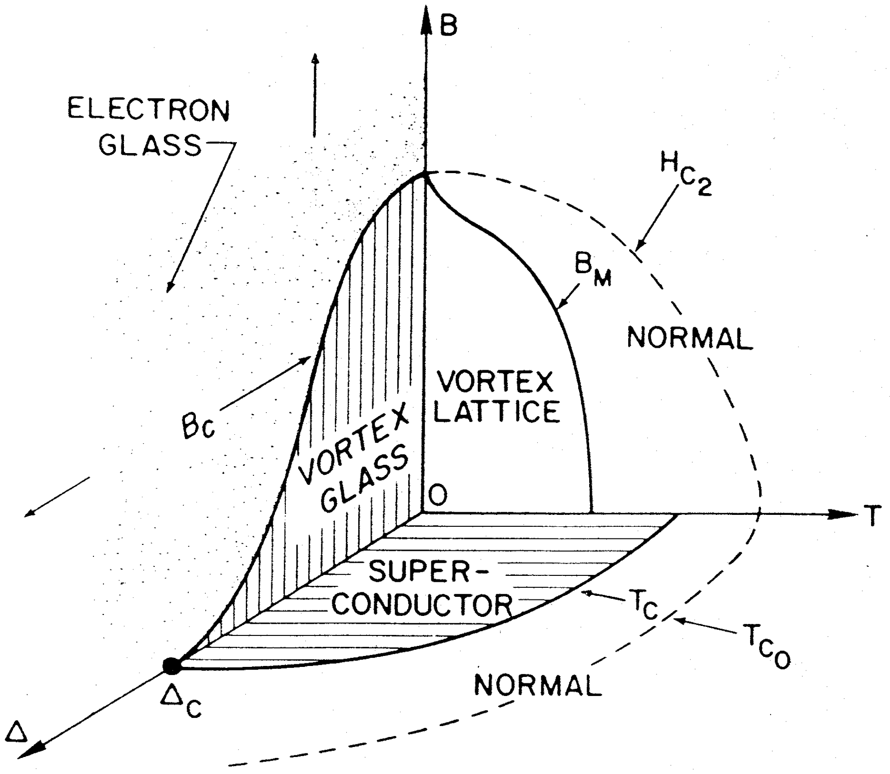
It is expected that for a field-driven transition, the critical field B_c scales as $B_c \sim T_c^{2/z}$. This enables independent measurement of dynamical exponent z.[8]

The values obtained are: $z \approx 1.02 \pm 0.05$, $z\nu \approx 1.26$.

3 Dirty Boson Model

The central assumption of the dirty boson model is the ignoring of fermionic degrees of freedom (Remember, fermions are always localized in 2D). Cooper pairs, are then viewed as composite bosons, interacting with each other, and moving in a random potential.

The transition between normal and superconducting states is associated with binding of the vortices. The Cooper pair order parameter is non-zero below T_{c0}, however, because of fluctuations in 2D, no phase coherence is established. If one goes below the KT transition temperature T_c, the vortices bind into pairs and the power-law order is established. This is sufficient for a system to exhibit superconducting behavior.[9]



Above is a schematic phase diagram. The parameters that tune transition(s) are the disorder Δ, magnetic field B and the temperature T.

$\Delta = 0$ plane does not need any explanation: it demonstrates the melting of the Abrikosov lattice.

In $B = 0$ plane, we can see that with increasing disorder, T_c vanishes before T_{c0}. At $T = 0$ we can go from superconducting to insulating phase because of vanishing T_c. That T_{c0} is non-zero implies that well-defined Cooper pairs exist on both sides of the transition — dirty boson model should correctly describe both phases.

Finally, in $T = 0$ plane for sufficiently low magnetic fields the vortices are localized by disorder and repulsive interactions. An applied magnetic field will delocalize the vortices and localize the electrons. (It is no clear, however, whether the magnetic field localizes Cooper pairs, or breaks them into individual electrons. In the former case, second critical field might be observed)

4 Duality

In the simplest case, dirty boson Hubbard model can be described by a Hamiltonian:

$$H = \frac{U}{2}\sum_i (n_i - n_0)^2 - \sum_i \mu_i (n_i - n_0) - t\sum_{\langle ij\rangle} b_i^\dagger b_j$$

$\frac{U}{2}$ is an on-site repulsion, μ_i is the random potential, and t is the hopping term.

Taking $b_i^\dagger = \sqrt{n_0} e^{i\phi_i}$, we obtain (approximately):

$$H = \frac{U}{2}\sum_j \left(\frac{1}{i}\frac{\partial}{\partial\phi_j} - n_0\right)^2 - \sum_j \mu_j \left(\frac{1}{i}\frac{\partial}{\partial\phi_j} - n_0\right) - J\sum_{\langle ij\rangle}\cos(\phi_i - \phi_j) \equiv \frac{U}{2}\sum_i \hat{n}_i^2 - \sum_i \mu_i \hat{n}_i - J\sum_{i,\nu}\cos(\nabla_\nu \hat{\phi}_i)$$

∇_ν denotes lattice derivative here. $\hat{n}_i$ is the deviation of occupation from the mean number. For integer n_0 and μ_i, this model is equivalent to $(2+1)$D XY model. The latter is known to be dual to a lattice superconductor with gauge field.

By a set of nearly exact duality transformations, the action can be cast into the following form:??

$$S(A,\theta) = S_0 + S_1$$

$$S_0 = \sum_i \int d\tau \{\frac{1}{2t}|(\nabla\times A)^\perp|^2 + \frac{u}{2}|(\nabla\times A)^\tau|^2 - \mu_i(\nabla\times A)^\tau\}$$

$$S_1 = \frac{1}{2K}\sum_i \int d\tau (A^\tau - \dot{\theta})^2 + J\sum_{i,\nu}\int d\tau \cos(\nabla_\nu\theta_i - A_i^\nu)$$

∇ is continuous in time dimension and discrete in space. $(\nabla\times A)^\tau$ is effectively the boson density.

Note that in contrast to phase representation, the broken gauge symmetry in dual representation ($\langle e^{i\theta}\rangle \neq 0$, finite mass (gap) for A) implies the destruction of superconductivity.

Let us now incorporate long-ranged interactions and rewrite everything in more symmetric form.

Again, the original hamiltonian is:

$$H = H_0 + H_1$$

$$H_0 = \frac{1}{2}\sum_{i,j} V_{ij}(\hat{n}_i - n_0)(\hat{n}_j - n_0) + \sum_i \mu_i \hat{n}_i$$

$$H_1 = -J\sum_{\langle ij\rangle}\cos(\hat{\phi}_i - p\hat{h}i_j + A_{ij}^{ext})$$

In dual representation:

$$H' = H'_0 + H'_1 + H'_2$$

$$H'_0 = \frac{1}{2}\sum_{ij} G_{ij}(\hat{N}_i - B)(\hat{N}_j - B)$$

$$H'_1 = -J'\sum_{\langle ij\rangle} \cos(\hat{\theta}_i - \hat{\theta}_j - A_{ij})$$

$$H'_2 = \frac{1}{2}\sum_{ij} V_{ij}(\nabla \times A_i - n_0)(\nabla \times A_j - n_0) + \sum_i \mu_i(\nabla \times A) + \sum_i |\Pi_i|^2$$

$\hat{N}_i$ is the vortex number operator conjugate to $\hat{\theta}_i$ and Π_i is the momentum conjugate to A. A^{ext} is the potential of external magnetic field.

If the problem were exactly self-dual, one would expect particle and vortex resistivities to be equal:

$$\tilde{\rho}_{\alpha\beta}(\delta) = \tilde{\sigma}^v_{\alpha\beta}(\delta).$$

(in units of $4e^2/\hbar$)

This is the resistance/conductance duality observed experimentally. The model is not exactly self-dual. Cooperons interact by $V_{ij} \sim \frac{1}{r}$, while vortices have $G_{ij} \sim -\ln r$ However, in some materials with high dielectric constants, the Cooper pair interaction is closer to logarithmic rather than Coulombic form. The system posesses vortex-antivortex symmetry, while particle-hole symmetry is absent for bosons. This constitutes another problem in attempts to apply duality to SI-transition.

Magnetic field breaks vortex-antivortex symmetry, leading one to suspect that magnetic-field driven transitions are closer to being self-dual.

5 Universal conductivity

An important experimental evidence of SI transition is the existence of universal conductivity. It has been observed that the normal state resistance corresponding to a border between superconductive and insulating behavior is close to the value of quantum unit resistance $R_Q = h/(2e)^2 \approx 6.45k\Omega$.

It was hypothesized that SI transition is in the same universality class as the dirty boson model. There exists a natural explanation of universal conductivity within duality picture.

The finite resistivity in film is caused by the flow of vortices. We imagine that a current in a square sample is described by the flow of Cooper pairs of the total flux $\dot{n}_c$. The correspondig current is $I = 2e\dot{n}_c$. Vortices moving perpendicular to the current give rise to the finite voltage, the value being given by Josephson relation $V = \frac{h}{2e}\dot{n_v}$. Hence the resistivity is $R = \frac{V}{I} = \frac{h}{(2e)^2}(\frac{\dot{n_v}}{\dot{n_c}})$ If we assume that bosons and vortices are dual to each other (that is, if in dual representation vortices are viewed as particles, the particles are their respective vortices), right at the transition one must have $\dot{n_c} = \dot{n_v}$, and has the universal resistance should be exactly equal to R_Q.[11]

For the field-tuned transition, which is believed to be "closer" to self-duality, the sum of squares of longitudinal and Hall resistances should be R_Q^2.

The universal conductivity has been calculated for (2+1)D XY model (which is equivalent to a pure boson Hubbard model at integer filling). The 1/N expansion yielded the following result:

$$\sigma^* = \frac{\pi}{8}(\frac{(2e)^2}{\hbar})(1 - \frac{1}{N}\frac{64}{9\pi^2}) \approx 0.251\frac{(2e)^2}{\hbar} at N = 2$$

Monte-Carlo simulations of the "soft-spin" version of $(2+1)$D XY model yield the result $\frac{\sigma^*}{\sigma_Q} = 0.285 \pm\ 0.02$

These results are not important for SI-transition, because the latter is believed to occur from Bose-glass phase rather that Mott-insulator phase. More results will be provided below in a section on RG studies.

6 Scaling

In this and following sections, the reader will be amused by a huge number of incompatible results. The critical behaviour of boson model depends drastically on whether the forces are short-ranged or long ranged, and whether the transition occurs from the Mott-insulator or Bose-glass. A transition from Bose-glass with long-ranged Coulomb forces is deemed relevant for superconductor-insulator transition, however all results worth to be mentioned here and compared with experiment.

In the absence of disorder, the transition occurs between the incompressible Mott-insulator and a superfluid. For arbitrarily weak disorder, a new gapless insulating phase should intervene. For sufficiently strong disorder Mott phase can be destroyed completely.[12]

To discuss the transition, one has to introduce a parameter, δ, which is analogous to the reduced temperature t for finite-temperature transitions and thus measures the distance from the transition. (At the transition $\delta = 0$). One can take for example:

$$\delta \sim \mu - \mu_c$$

with μ_c the chemical potential at the transition.

The ususal assumption is that the transition is governed by a single characteristic length ξ, which diverges as $\delta^{-\nu}$ at criticality (this defines ν)

For $T = 0$ (quantum) phase transition, both static and dynamic quantities are linked. One has to introduce a characteristic frequency Ω. Ω is expected to vanish algebraically at the transition:

$$\Omega \sim \xi^{-z} \sim \delta^{z\nu}$$

, which defines the dynamical exponent z.

The need for a time-scale can be seen from the partition function:

$$Z = \int \prod_i D\Phi_i(\tau) D\Phi_i^* e^{-S/\hbar}$$

$$S = \int_0^{\beta\hbar} d\tau \sum_i \Phi_i^* \hbar \partial_\tau \Phi_i + \int_0^{\beta\hbar} d\tau \ H(\Phi^*, \Phi)$$

fields $\Phi_i(\tau)$ satisfy $\Phi_i(\tau = 0) = \Phi_i(\tau = \beta\hbar)$.

The diverging time scale Ω^{-1} can be thought of as a "correlation length" in the time dimension.

The homegeneity condition near the transition writes as:

$$f_s(\delta) \sim \ b^{-(d+z)} f_s(b^{1/\nu}\delta) \sim \delta^{\nu(d+z)}$$

The order-parameter susceptibility (averaged over disorder realizations) has the folowing scaling form:

$$\bar{\chi}(r,\tau) \sim \ r^{-(d+z-2+\eta)} g(r/\chi, \tau/\chi^z)$$

It is convenient to add to the action the external source (field), coupled to Φ's:

$$S \rightarrow \ S - \int \ d\tau \sum_i [h\Phi_i^*(\tau) + c.c.]$$

This external field explicitly breaks the symmetry and destroys the transition, whence must be relevant. One expects the energy density to scale as:

$$f_s(\delta, h) \sim \delta^{\nu(d+z)} Y(h/\delta^{\phi_h})$$

.

Using the fact that $\bar{\chi}(k = 0, \omega = 0) \sim \partial^2 f/\partial h^2$ one can relate $\beta(M \sim \delta^\beta)$ and η:

$$2\beta = \nu(d + z - 2 + \eta)$$

This relation can also be rewritten in a more familiar form:

$$\alpha + 2\beta + \gamma = 2$$

The natural generalization of the specific heat to quantum phase transitions would be the compressibility (provided $\delta \sim \mu - \mu_c$).

$$\kappa \equiv -\partial^2 f/partial\mu^2 \sim -\partial^2 f/\partial\delta^2$$

(compare to $c \sim \partial^2/\partial\ t^2$ for finite-temperature PT)

The *singular* part of the compressibility, κ_s, should scale as:

$$\kappa_s \sim \delta^{\nu(d+z)-2} \equiv \delta^{-\alpha}$$

(Compare to hyperscaling law $d\nu = 2 - \alpha$ for $T \neq 0$)

At $T = 0$ the order parameter depends on both space and imaginary time. The long-wavelength, low-energy effective action describing the superfluid phase is expected to have the following form (expansion in powers of $\dot{\phi}$ and $\nabla\phi$)

$$S_{eff} = \frac{1}{2}\int\ d^dx \int_0^{\beta\hbar} d\tau[2\rho(i\hbar\dot{\phi}) + \kappa(\hbar\dot{\phi})^2 + (\hbar^2\rho_s/m)(\nabla\phi)^2]$$

Scaling forms for superfluid density and compressibility are:

$$\rho_s \sim \delta^\zeta, with\zeta = \nu(d+z-2)$$

$$\kappa \sim \delta^{\nu(d-z)}$$

In the absence of disorder the transition occurs from the incompressible Mott state. There is no analytic piece of κ, hence the exponent for the singular part is the same as that for the total compressibility.

Solving algebraic equation one obtains $z\nu = 1$. Above the critical dimension the hyperscaling laws cease to hold and the exponents ζ and ν aquire their mean-field values of 1 and $\frac{1}{2}$ respectively. This enables to determine $d_c = 2$

The central assumption in the following is the finiteness of compressibility κ right at the transition (it is finite in superfluid and Bose-glass phase).[12]

Indeed if $z > d$ κ were to diverge at the transition, $\kappa = \kappa_s$, and both $z\nu = 1$ and $\nu \geq 2/d$ could not be satisfied.

If $z < d$, then $\nu = 1/z \geq 2/d$, hence $z \leq\ d/2$.

The second sound speed ($\sim\ \delta^{\nu(z-1)}$ must not diverge), so we are led to conclude that $z \geq 1$.

The only remaining possibility is $z = d$

Now consider the case of long-ranged interactions.[13] The action for a system of bosons is:

$$S = S_0 + S_1$$

$$S_0 = \int\ d^dxd\tau[\psi^*\partial_\tau\psi + (\hbar/2m)|\nabla\psi|^2 + U(x)|\psi(x,\tau)|^2],$$

$$S_1 = \int\ d^dxd^dx'd\tau\ V(x-x')[|\psi(x,\tau)|^2 - n_0][|\psi(x',\tau)|^2 - n_0]$$

Here $V(x) = (2e)^2/|x|$ - the repulsion between bosons, $U(x)$ - random potential. At fixed disorder, there exists a critical density n_c, above which system goes from Bose glass phase to a superconducting phase.

As usual two central critical exponents are $\nu : \xi \sim \delta^{-\nu}$ and $z : \Omega \sim \xi^{-z}$

In the superfluid phase the second sound mode has plasmon-like dispersion relation. The effective phase-only action is:

$$S_\phi = \frac{1}{2}\int d^dkd\omega[(\rho_s\hbar/2m)k^2 + \hbar\omega^2|k|^{d-1}/e_R^2]|\phi(k,\omega)|^2$$

where ρ_s is the renormalized superfluid density and e_R is the "renormalized" charge,

$$e_R^2 \equiv \lim_{k\to\ 0} |k|^{d-1}/chi_{\rho\rho}(k,\omega=0)$$

One can immediately see from this expression that ρ_s should scale as $\frac{1}{\xi^2}\xi^{-d}\Omega^{-1} = \xi^{-(d+z-2)}$

Similarly, the renormalized charge should scale $e_R^2 \sim \xi^{1-z}$ In a charged system e_R cannot be zero. On the other hand, provided that the insulating phase is a gapless Bose glass, e_R cannot be infinite (it is infinite in Mott state)

Hence we draw a conclusion that $z = 1$ so that e_R remains finite. The scaling form for conductivity is:

$$\sigma(\omega) = (2e)^2\rho_s(-i\omega)/(-im\omega)$$

, where

$$\rho_s(\omega) = \xi^{-d}(\xi/a)^{2-z}\tilde{\rho}_s(\omega/\Omega)$$

$$\tilde{\rho}_s(x) \to\ const as x \to\ 0$$

$$\tilde{\rho}_s(x) = c_d x^{(d+z-2)/z} as x \to \infty$$

So $T = 0$ conductivity is a finite constant, $c_d e^2/\hbar$. At criticality $\sigma(\omega, \xi = \infty) \sim \omega^{(d-2)/z}$ or $\sigma(T, \xi = \infty) \sim\ T^{(d-2)/z}$

7 RG studies

The most comprehensive analysis of superfluid to Bose glass transition has been given by I.F. Herbut. He performed an expansion in $\epsilon = d - 1$ around lower critical dimension of 1.[14]

Short-range repulsion.

Starting from the expression for the effective action determined in previous section, after introducing replicas and averaging over the random potential (It is

assumed to have a Gaussian probability distribution $P[U(k)] \sim \exp(-\frac{U^*(k)U(k)}{D})$
)

one obtains:

$$S_\theta = K \sum_{i=1}^{N} \int dx \int_0^\beta d\tau (c^2(\partial_x \theta_i)^2 + (\partial_\tau \theta_i)^2) - D\rho_0^2 \sum_{i,j=1}^{N} \int dx d\tau\, d\tau \cos 2(\theta_i(x,\tau) - \theta_j(x,\tau))$$

with a new field θ_i

Here $K = \frac{m}{2\pi^2 \hbar \rho_s}$ - superfluid density, and $c^2 = \frac{\rho_s}{\kappa m}$ - second sound velocity

The recursion relations (s is the length scale):

$$\frac{dK}{d\ln(s)} = (2 - z - d)K + \frac{8D}{Kc^4} + O(D^2)$$

$$\frac{dc}{d\ln(s)} = (z - 1 - \frac{4D}{K^2c^4})c + O(D^2)$$

$$\frac{dD}{d\ln(s)} = (d + 2z - \frac{1}{Kc})D + O(D^3)$$

Solving these equations for the fixed point one obtains(c=1):

$$K^* = \frac{1}{3} - \frac{\epsilon}{3} + O(\epsilon^2)$$

$$D^* = \frac{\epsilon}{36} + O(\epsilon^2)$$

$$z = 1 + \epsilon + O(\epsilon)$$

It can be proved rigorously that $z = d$ to all orders in ϵ.

From linearized group flow equations one also obtains:

$$\nu = \frac{1}{\sqrt{3\epsilon}} + O(\sqrt{\epsilon}).$$

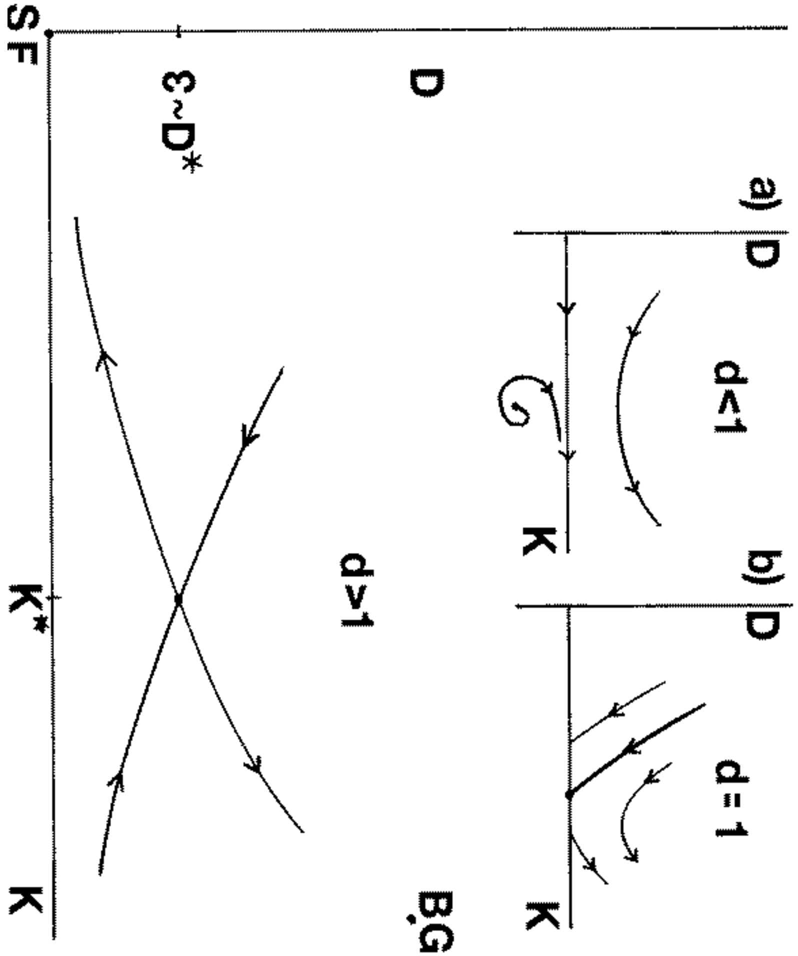
The correlation function decays as $\bar{G(x)} \sim \rho_0 |x|^{-K^* c^*}$.

From here the anomalous dimension η evaluates to $\frac{1}{3} - \frac{7\epsilon}{3} + O(\epsilon^2)$.

One may also want to include higher-order terms in expansion of the action in ϕ and Π (conjugate to ϕ):

Simple dimensional analysis tells us that all higher-order terms are irrelevant as long as $z = d$ with the exception of the term proportional to $\Pi(x)|\nabla\phi(x)|^2$. If the coupling constant is denoted by v_x, the recursion relation for it is:

$$\frac{dv_x}{dln(s)} = (z - 2 - \frac{16D}{K^2c^4} + O(D^2))v_x + O(v_x^2)$$


a)
D
d<1
K
b)
D
d=1
K
D
ε~D*
SF
d>1
K*
K
B.G

The case of z=d=2 is special. Strictly speaking it is not enough to retain the first term in D only because $D^*/K^{*2} \sim 1$. Nevertheless, most likely v_x turns out to be irrelevant even for $d=2$.

For the coupling constants D_m of higher-order harmonics of random potential ($\Delta\ S \sim \cos 2m(\theta_i - \theta_j)$), the recursion relations read

$$\frac{D_m}{d\ln(s)} = (d + 2z - \frac{m^2}{Kc})D_m + O(D_n^2)$$

D_m's turn out to be irrelevant for $m \geq 2$

Coulomb repulsion.

Only the recursion relation for c is modified:

$$\frac{dc}{dln(s)} = (z + \frac{d-3}{2} - \frac{4D}{K^2c^4})c + O(D^2)$$

Now $c^2 = 4\pi\hbar\rho_s e^2/m$ And the critical exponents are:

$$z = 1$$

$$\nu = \sqrt{\frac{2}{3\epsilon}} + O(\sqrt{\epsilon})$$

$$\eta = \frac{1}{3} - \frac{10\epsilon}{9} + O(\epsilon^2)$$

These calculations allow one to establish exact values for the dynamical exponent z, the values of ν and η are not reliable; moreover ν fails to obey Harris criterion $\nu > 2/d$ for disorder tuned transitions.

There exists a calculation for BG-SF transition (short-range repulsion). The author used RG at fixed dimension, the following values for critical exponents (to lowest order in FP values of coupling constants) are obtained:[15]

$$z \approx 1.93$$

$$\nu \approx 1.38$$

The universal conductivity has been found to equal $\sigma_c \approx 0.25(2e)^2/\hbar$.

Conclusion

Superconductor-insulator transitions are believed to be in the same universality class as the SI transitions in Josephson junction arrays, He in disordered media, vortex transitions in type-II superconductors in magnetic field in presence of correlated disorder. RG results can be directly applicable to other models, and the theoretical predictions can be tested there.

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