Random Field Ising Model

Markus Dittrich

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Abstract

This paper gives an introduction to the *Random Field Ising Model (RFIM)*. Since its first discussion in the paper by Imry and Ma [1] there has been great interest in this model, since Ising or Ising-like systems in random fields are a good representation of a large number of impure materials. These show features that can not be understood by studying ideal systems (i.e. Ising model). There are a lot of interesting topics related to the RFIM e.g. the stability of the ferromagnetic phase and lower critical dimension d_l, interface properties, order of the phase transition, scaling laws, renormalisation group treatment and crossover phenomena. We will then more specifically discuss the $3d$ RFIM that is currently under strong investigation, in particular the problem of characterizing the phase transition. Since diluted antiferromagnets are a convenient probe for studying random fields in an experimentally controlable way (and have been shown to belong to the same static critical behaviour universality class) they will be discussed in some detail. Experimental data will be presented to compare with the theoretical predictions made.

1 Introduction

Before I start discussing the RFIM I would like to warn the reader right at the beginning, especially if he plans to go deeper into the subject. The RFIM has first been proposed about twenty years ago by Imry and Ma [1] and up to the present day, even though a lot of progress has been made, many of its features are either not yet understood or there are different opinions about what the actual physical properties are. The physical picture I will present in this paper represents the level of understanding in 1997 and there might be substantial changes in the next years. There are of course properties that are believed to be understood and should therefore survive further progress in the subject.
Before we take a closer look at the RFIM it is worthwhile to review the Ising model itself. It is described by the well known Hamiltonian

$$H = -\sum_{ij} J_{ij} S_i S_j - \sum_i H_i S_i \tag{1}$$

where $S_i = \pm 1$ are spins in a d-dimensional hypercubic lattice. The Ising model exhibits spontaneous symmetry breaking and one usually assumes the coupling between the spins to be translational invariant, i.e. $J_{ij} = J(|\boldsymbol{r}_i - \boldsymbol{r}_j|)$, $\boldsymbol{r}_i, \boldsymbol{r}_j \in \Lambda^d$ and the external fields H_i are typically homogeneous, $H_i = H \ \forall i$. Even under those simplifying assumptions the Ising model may show very complex behaviour.
Several centuries ago people began to realize the importance of quenched impurities in solids. All solid materials contain some quenched randomness in form of defects, impurities or as a general structural property. In some materials the randomness is an integral part of their characteristics. Even surprisingly small amounts of quenched randomness can greatly influence the phase transition. Specific interest was devoted to the effect of impurities on order-disorder transitions. The randomness causes real physical systems to be never completely translational invariant. Hence in order to compare experimental data, e.g. critical exponents with predicted theoretical values the pure models (e.g. Ising model) are often not appropriate. In order to obtain a more realistic description the Hamiltonian of the Ising model has to be modified in the following way

$$\begin{aligned} J(|\boldsymbol{r}_i - \boldsymbol{r}_j|) &\rightarrow J(|\boldsymbol{r}_i - \boldsymbol{r}_j|) + \delta J_{ij} \\ H &\rightarrow H + h_i \end{aligned} \tag{2}$$

where J_{ij} and h_i are random quantities taking into account the disorder present in real samples. Those random quantities are characterized by some probability distribution. Typically J_{ij} and h_i have zero average for different bonds and are uncorrelated for different sites respectively. There are a few cases to distinguish. In the case that $\delta J_{ij} \ll J_{ij}$ there are no drastic effects. Some critical exponents for second order transitions may change and first order transitions can become second order. In the other limit $\delta J_{ij} \gg J_{ij}$ LRO is generally destroyed and a spin glass phase may occur. The topic of this paper, however, is the case

$$\delta J_{ij} = 0 \quad , \quad h_i \neq 0 \tag{3}$$

The Ising Hamiltonian with the changes given by equations 2 and 3 is called the Random Field Ising Hamiltonian, i.e. in absence of external magnetic fields

$$H = -\sum_{<ij>} J_{ij} S_i S_j - \sum_i h_i S_i \tag{4}$$

where we have restricted the interactions to nearest neighbour only. This Hamiltonian describes the competition between random ordering fields and long range order. The second term is not present in the pure Ising model. The J_{ij} are positive and the ground state in the absence of fields is ferromagnetic. In order to obtain specific results we have to specify the h_i. In the following the notations $< ... >$ and the overbar designate a thermal average and a disorder average over the random fields respectively. The probabilty distribution for the disorder average is typically a Gaussian or a bimodal one, where the latter one is given by $h_i = \pm h_r$ with equal probability and h_r being some constant value of the field. We make the following assumptions about h_i

$$\overline{h_i} = 0 \quad , \quad \overline{h_i h_j} = h_r^2 \delta_{ij} \quad , \quad h \ll J \tag{5}$$

The general ingredients in the RFIM Hamiltionian that are important for our purposes are the short range interactions and the assumption of one dimensional spins. In real magnetic systems the last assumption is often only approximately true, one at most has strong anisotropy, which can make the spins quasi one dimensional. Even though we call the S_i in our Hamiltonian spins for brevity the system itself is not restricted to magnetism. The S_i, for example, could also describe a molecule with two different orientations, occupied or empty sites in a lattice or a two state microscopic system. In general the RFIM is appropriate to describe any solid state system that has a transition with a doubly degenerate ordered state and contains frozen impurities or point defects that cannot move on the relevant time scales. Another convenient way to describe the system is through the Ginzburg-Landau Hamiltonian

$$H_{GL} = \int d^d r \left[\frac{1}{2} \tilde{a} \phi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{4} u \phi^4 - h(\boldsymbol{r}) \phi(\boldsymbol{r}) \right] \tag{6}$$

with $-\infty < \phi(\boldsymbol{r}) < \infty$, $\tilde{a} = a'(T - T_c(0))$, $T_c(h=0) = O(J)$ and $\overline{h(\boldsymbol{r})} = 0$, $\overline{h(\boldsymbol{r}) h(\boldsymbol{r}')} = h^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$.

As mentioned above, the RFIM was first discussed in 1975 in a paper by Imry and Ma [1] and is currently under strong theoretical and experimental investigation. The experimental progress in the RFIM, however, came not before it was recognized that dilute antiferromagnets in homogeneous fields are ideal experimental probes to study these systems. We will discuss this in a later section. There are

other realisations of the RFIM in nature such as adsorbed mono-layers on impure substrates, cooperativity of protein folding and binary liquids in porous media.
In the rest of the paper I try to work out what is new about the RFIM, i.e. what properties are different from those of the well known pure Ising model.

2 Stability of ferromagnetic phase

This important topic was first addressed in [1]. We know that the Ising model with Hamiltonian given by equation 1 and $H = 0$ exhibits long range order for $d > 1$, or in other words it has a lower critical dimension $d_l = 1$. Lower critical dimension of course refers to the dimension above which long range order occurs. It is obvious that a random field term given by $h_i = -\sum_i h_i S_i$ will act against this ordering. The interesting case is not $h \gg J$, where it is clear that the system will be disordered, but the case $h \ll J$. The question we want to answer is: is there any change in the lower critical dimension d_l, or in other words, is there a tendency due to the disorder, to suppress the ordering in lower dimensions and hence to shift d_l to higher values? Imry and Ma showed in their paper that the ferromagnetically ordered state becomes unstable with respect to the formation of ill oriented domains in all dimensions $d \leq 2$ so that $d_l = 2$. The argument they gave and that is known as the *Imry-Ma argument* is the following: We regard a domain of linear dimension R. The energy cost for reversing the spins in this domain is proportional to the domain wall area, hence

$$E_{ex} \propto JR^{d-1} \tag{7}$$

where we have set the lattice constant a equal to one. For low temperatures and vanishing random fields, this is obviously not favourable so that there will be no spontaneous formation of domains. The situation, however, changes if there are random fields present. There is an energy gain due to the interaction with these random fields. The energy is given inside a region of volume R^d by

$$E_R^2 \propto h^2 R^d \tag{8}$$

E_R itself can be positive or negative. It is, however, always possible to find a region about an arbitrary lattice point i such that $E_R > 0$. Hence, reversing the spins in this domain results in an energy gain of $-2E_R$. We obtain for the total energy of a domain of size R in the presence of random fields

$$E(R) \propto JR^{d-1} - hR^{\frac{d}{2}} \tag{9}$$

It is now obvious that for $h \ll J$

$$\begin{aligned} E_R &> 0 \quad \text{for} \quad d > 2 \\ E_R &< 0 \quad \text{for} \quad d < 2 \quad \text{for large enough } R \end{aligned}$$

The ferromagnetic state is therefore unstable for $d < 2$. In the case $d=2$, it is not a priori obvious, but it has been shown [10] that there exists an instability for $d=2$ due to the roughness of the domain wall surface (see next section) so that there is no long range order in two dimensions. The lower critical dimension therefore changes to $d_l=2$. Hence we obtain the surprising fact that even for arbitrary week random fields, long range order is destroyed in $d\leq 2$ and the lower critical dimension changes from $d_l = 1$ to $d_l = 2$. It has been rigorously proven that in dimension $d = 3$ there exists LRO for weak random fields and low temperatures. It should be understood that the above condition is only sufficient for the nonexistence of the ordered state. There might be stronger instabilties that we will neglect in the following. In addition, there are other complications that we have not taken into account in our considerations, e.g.

- domains within domains $\rightarrow$ it can be shown that they are rare for $d>2$ [14]
- there generally exist a large number of different contours for a domain of size R $\rightarrow$ it can be shown that they all basically include the same volume and random fields and that with probability one there is no contour enclosing a random field gain that is larger than the surface energy loss if $d>2$
- thermal fluctuations $\rightarrow$ those are supposed to be irrelevant at low temperatures

Concluding this section we found that the lower critical dimension changes from $d_l=1$ in the Ising model to $d_l=2$ in the RFIM.

3 Interface Properties

Applying the Imry-Ma argument in the preceeding section we have implicitly assumed that domain walls are smooth, such that their surface area is in fact proportional to R^{d-1} if R is the linear dimension of the domain wall. It is therefore in order to check if this assumption is justified. We consider a domain wall with vanishing mean curvature, e.g. a small part out of the surface of a larger domain. We denote the surface profile by $z(\underline{x})$ where $\underline{x}$ is a $(d-1)$-dimensional vector parallel to the mean interface plane, i.e. $\boldsymbol{r} = (\underline{x}, z)$. We define the interface roughness by

$$w(L) = < \overline{(z(\underline{x}_1) - z(\underline{x}_2))^2} >^{\frac{1}{2}} |_{L=|\underline{x}_1 - \underline{x}_2|} \propto L^{\zeta} \tag{10}$$

If $\zeta \neq 0$ but $\zeta < 1$ the interface is called self-affine. With help of an Imry-Ma argument it can be shown that

$$w(L) \approx \left(\frac{Mh}{\Gamma}\right)^{\frac{2}{3}} L^{\frac{5-d}{3}} \tag{11}$$

where Γ is the wall stiffness. The so called roughness exponent $\zeta = (5 - d)/3$ is easily seen to be

- smaller than 1 for $2 < d < 5$ and one can conjecture that domain walls are essentially flat in this case
- smaller than 0 for $d > 5$ and walls are always flat in this case

For $d > 2$ we see that wall gradients ($\propto L^{\zeta-1} \propto L^{(2-d)/3}$) are always small and hence walls are smooth on large scales. On the other hand we expect domain walls to be rough in $d = 2$ dimensions and below. We can therefore conclude that the results we obtained by the Imry-Ma argument in the preceeding section are correct. In addition we worked out the wall roughness that is responsible for the destruction of LRO in two dimensions as mentioned above.

4 Phase Transition

Since we have established that the lower critical dimension of the RFIM is different from that of the Ising model, we have to address the question of characterizing the phase transition that should occur in dimensions 3 and greater. In the pure Ising model we have a continuous transition in $d = 3$. For the RFIM *mean field theory* predicts that as the random field is increased from zero, the transition temperature decreases until it goes to $T_c = 0$ at a critical field h_r^c. For $h_r > h_r^c$ the system is disordered at all temperatures. For random field probability distributions that monotonically decrease with increasing field $|h|$ (e.g. Gaussian distribution), the transition (in mean field theory) is always continuous. For the bimodal distribution there exists a tricritical point, i.e. the transition becomes first order for larger critical fields. Recent Monte Carlo simulations for bimodal and Gaussian distributions, however, do not show latent heat or phase coexistence but a jump in M. On the other hand, high temperature expansions show continuous transitions for both distributions. This shows that so far no proof exists for a continuous transition in either case. Nevertheless it will be assumed, at least for the Gaussian distribution, that there is a continuous transition. This seems to be justificable.

5 Scaling laws

In this section we take a closer look at the critical exponents that establish a means of characterizing the critical behaviour. Before we start it is probably convenient to write down the definitions of the exponents and some of the known scaling laws for the ideal system.

$$
\begin{array}{ll}
C_p \propto |t|^{-\alpha} & \alpha + 2\beta + \gamma = 2 \\
\chi \propto |t|^{-\gamma} & \beta\delta = \beta + \gamma \\
\xi \propto |t|^{-\nu} & 2 - \alpha = \nu d \quad \text{hyperscaling law} \\
M \propto |t|^{\beta} \quad (t < 0) & \nu(2 - \eta) = \gamma
\end{array}
$$

The important question is: Are random fields important at the critical point? If yes, under what conditions?
Consider first the system at the true transition temperature $T_c(H)$ in a finite homogeneous field H. We divide the system into blocks of linear size ξ, where ξ is the correlation length. By standard scaling theory

$$H \propto J\xi^{\frac{-\beta\delta}{\nu}} \tag{12}$$

It is generally assumed that random fields produce an additional excess field

$$\delta H \propto ch\xi^{-d/2} \tag{13}$$

where c is a constant of $O(1)$. We now want to obtain the condition for when we can expect a sharp transition. If we approach the critical point through $H \to 0$ we expect a sharp transition if

$$\frac{\delta H(\xi)}{H(\xi)} \to 0 \qquad \text{for} \qquad \xi \to \infty \qquad \text{i.e.} \qquad \frac{d}{2} \geq \frac{\beta\delta}{\nu} \tag{14}$$

This condition, however, is only applicable if $\xi < \xi_{LG}$, where ξ_{LG} is determined by the Landau-Ginzburg criterion. Otherwise the linear response expression for δH is not longer valid. From the scaling laws above, namely $\alpha + 2\beta + \gamma = 2$ and $\beta\delta = \beta + \gamma$ we obtain

$$\frac{d}{2} \geq \frac{2 + \gamma - \alpha}{2\nu} \tag{15}$$

where we have assumed that the scaling relations are still valid in the random field case. If we insert mean field exponents

$$\beta = \nu = \frac{1}{2} \;\; , \;\; \delta = 3 \;\; , \;\; \gamma = 1 \;\; , \;\; \alpha = 0 \tag{16}$$

we obtain the result $d \geq 6$. Random fields in this approximation are only negligible for $d \geq d_c = 6$ and are therefore relevant for the phase transition in $d < 6$. We know from the ideal Ising system that for $d < 4$, we already have non-classical exponents (i.e. mean field theory is not correct). With help of the Josephson scaling law $\nu^{(0)} d = 2 - \alpha^{(0)}$ where the superscript denotes the nonclassical exponents for the pure system, we obtain

$$\frac{d}{2} \geq \frac{2 + \gamma^{(0)} - \alpha^{(0)}}{2\nu^{(0)}} \quad \Rightarrow \quad \gamma^{(0)} \leq 0 \tag{17}$$

Since $\gamma^{(0)} \geq 0$ per definition, there is obviously something wrong with our assumptions. We conclude that random fields are a relevant perturbation when we use the exponents of the pure model and that the conventional hyperscaling laws break down in random fields. In our case, this would be the Josephson scaling law. We instead have to assume a so called *modified hyperscaling law*

$$\nu(d - \theta) = 2 - \alpha \tag{18}$$

where we have introduced a new exponent θ. Using equation 15 we obtain the following relation for our exponent θ

$$\theta \geq \frac{\gamma}{\nu} = 2 - \eta \tag{19}$$

This has in fact already been proven for a number of random field systems by Schwartz and Soffer [17]. We see that there is a drastic change in the scaling behavior due to random fields. We know that in the pure system there are only two independent exponents which are related through the scaling laws. In the random field case we have to introduce a third independent exponent in order to avoid inconsistencies as for example a negative γ. Furthermore, all the hyperscaling laws change by replacing d through $(d - \theta)$. Some people claim that inequality 19, the so called *Schwarz-Soffer inequality*, holds as an equality. In this case we would again have only two independent exponents. But there is, however, no rigorous proof so far. The appearence of a third exponent is related to the existence of two different correlation functions that scale independently at $T = T_c(h)$, namely

$$\begin{aligned} G(\vec{k}) &= \overline{<S_{\vec{k}} S_{\vec{k}}>} - \overline{<S_{\vec{k}}><S_{-\vec{k}}>} \propto k^{-2+\eta} \\ G_{dis}(\vec{k}) &= \overline{<S_{\vec{k}}><S_{-\vec{k}}>} \propto k^{-4+\bar{\eta}} \end{aligned}$$

where the $S_{\vec{k}}$ are the Fourier components of the structure factor. G_{dis} obviously vanishes for the ideal system. In mean field theory one has $\eta = \bar{\eta} = 0$ for the pure system but if random fields are present, this changes to

$$\theta = 2 - \bar{\eta} + \eta \tag{20}$$

The Schwarz-Soffer inequality can then be rewritten as

$$\bar{\eta} \leq 2\eta \tag{21}$$

If the Schwarz-Soffer inequality would hold as an equality this would yield $\bar{\eta} = 2\eta$. The problem of actually having three instead of two independent exponents in the presence of random fields is not settled yet, but simulations suggest that the Schwarz-Soffer inequality might actually hold as an equality(c.f. figure 2, column 4).

6 Renormalisation Group

In this section I will give a short presentation of the renormalisation group (RG) treatment of the RFIM. Unfortunately there exists no satisfying RG analysis so far. In this section we assume a continuous transition up to $T = 0$. The singular part of the free energy can be written as

$$f_s = Jf\left(\frac{T}{J}, \frac{h}{J}, \frac{H}{J}\right) \tag{22}$$

We then carry out the RG coarse graining transformation with length scale factor l, corresponding to a reduction in the number of degres of freedom by l^d. The RG flow diagram (that also represents the phase diagram) for this system is expected to look like figure 1. The critical behavior of the pure system is controlled by the thermal

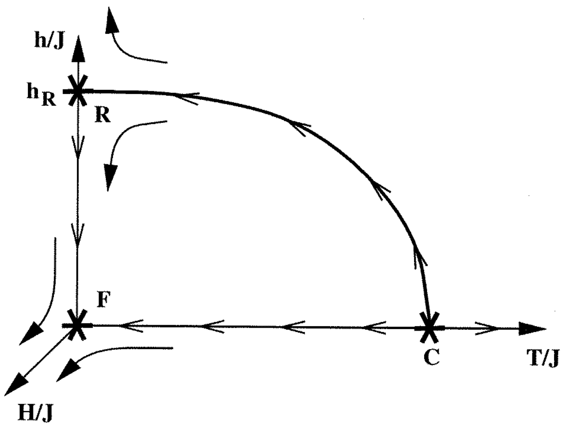

Figure 1: RG flow diagramm

fixed point C. Since the random fields are a relevant perturbation (the h_i break translational symmetry and we know that symmetry breaking terms are generally relevant operators), flows along the phase boundary between ferro and paramagnetic state approach the zero temperature random field fixed point R, which therefore controls the critical behaviour at the phase boundary. The flows starting in the ferromagnetic region terminate at the stable ferromagnetic sink F. With help of phenomenological arguments one obtains the following form for the scaling fields

$$\frac{T}{J}\,,\ \tau = \frac{1}{J}(h - h_R) + c\frac{T}{J} \quad \text{and} \quad \frac{H}{J} \tag{23}$$

Close to the fixed point R, our fields transform according to

$$\begin{aligned} J &\to J' = J l^{y_J} \\ \tau &\to \tau' = \tau l^{y_\tau} \\ H &\to H' = H l^{y_H} \end{aligned}$$

For the coarse grained free energy we obtain

$$\begin{aligned} f_s' &= l^d f_s\left(\frac{T}{J}, \tau, \frac{H}{J}\right) = J' f\left(\frac{T'}{J'}, \tau', \frac{H'}{J'}\right) \\ \Rightarrow f_s &= l^{y_J - d} J f\left(\frac{T}{J} l^{-y_J}, \tau l^{y_\tau}, \frac{H}{J} l^{(y_H - y_J)}\right) \end{aligned}$$

For the coarse grained correlation length

$$\begin{aligned}\xi' &= l^{-1}\xi\left(\frac{T}{J},\tau,\frac{H}{J}\right) = \xi'\left(\frac{T}{J}l^{-y_J},\tau l^{y_\tau},\frac{H}{J}l^{(y_H-y_J)}\right)\\ \Rightarrow \xi &= l\xi\left(\frac{T}{J}l^{-y_J},\tau l^{y_\tau},\frac{H}{J}l^{(y_H-y_J)}\right)\end{aligned}$$

If we make the choice $l \propto \tau^{-\left(\frac{1}{y_\tau}\right)}$ we obtain

$$\xi = \tau^{-\left(\frac{1}{y_\tau}\right)}\xi\left(\frac{T}{J}\tau^{\left(\frac{y_J}{y_\tau}\right)},\frac{H}{J}\tau^{-\left(\frac{y_H-y_J}{y_\tau}\right)}\right) \tag{24}$$

hence $\nu = \frac{1}{y_\tau}$. Since we now $f_s \propto t^{2-\alpha}$ and

$$f_s = \tau^{-\left(\frac{y_J-d}{y_\tau}\right)} f\left(\frac{T}{J}\tau^{\left(\frac{y_J}{y_\tau}\right)},\frac{H}{J}\tau^{-\left(\frac{y_H-y_J}{y_\tau}\right)}\right) \tag{25}$$

we obtain $2-\alpha = -\left(\frac{y_J-d}{y_\tau}\right) = (d-y_J)\nu$ which is nothing more than the modified hyperscaling law if we identify $y_J \equiv \theta$. The other critical exponents can be obtained as usual and are

$$\begin{array}{ll} \beta = \nu(d-y_H) & \nu = \frac{1}{y_\tau} \\ \gamma = \nu(2y_H - y_J - d) & \eta = 2 + d + y_J - 2y_H \\ \delta = (y_H - y_J)/(d-y_H) & \bar{\eta} = 4 + d - 2y_H \\ \theta = y_J & \end{array}$$

Hence we obtain the relations we already encountered in the preceeding section. The results obtained by real space RG among others are shown in figure 2.

7 Dimensional Reduction

The purpose of this section is to show in what way some of the confusion in treating the RFIM came about and how some of it has been removed only in recent years. A convenient starting point is the Ginzburg-Landau-Hamiltonian, equation 6. It can be shown that its most singular terms in *perturbation theory* follow from an iterative solution of the saddle point equation

$$\frac{\delta H_{GL}}{\delta\phi} = -\nabla^2\phi + V'(\phi) - h(r) = 0 \quad \text{where} \quad V(\phi) = \frac{1}{2}\tilde{a}\phi^2 + \frac{1}{4}u\phi^4 \tag{26}$$

This can be shown to lead to an exact relation between the critical exponents $\lambda_i^{RF}(d)$ (where λ corresponds to $\alpha,\beta,\gamma,\delta...$) of the RF-system in d-dimensions and those of the pure model in $(d-2)$-dimensions $\lambda_i^{(0)}(d-2)$

$$\lambda_i^{RF}(d) = \lambda_i^{(0)}(d-2) \tag{27}$$

Reference	$\theta = y_J$	$\nu = 1/y_T$	$\beta = \nu(d - y_H)$	$2\eta - \bar{\eta} = \theta - 2 + \eta$
MFA [a]	$d - 4$	$1/2$	$1/2$	$d - 6$
PT [b] $d = 6 - \epsilon$	2	$\nu^{(0)}(d-2)$	$\beta^{(0)}(d-2)$	$\eta^{(0)}(d-2)$
$d \to 2$	1	0	2	0
weighted MFA [58]	1.51	1.25 ± 0.11		
MC [c] (d=3)				
Rieger and Young [41]	1.56 ± 0.1	1.6 ± 0.3	0.003 ± 0.05	0.12 ± 0.12
Rieger [60]	1.53 ± 0.1	1.1 ± 0.2	0.00 ± 0.05	-0.03 ± 0.15
HTSE [d] Gofman et al. [42]	$(\gamma = 2.1 \pm 0.2)$			0
Realspace RNG				
Dayan et al. [52]	1.56	1.39	≈ 0	0.12
Newmann et al. [53]	1.00 ± 0.05	1.49 ± 0.008	1.66 ± 0.01	1.24
Cao and Machta [54]	1.5	$\nu = 2.25$	0.02	0.02
Falicov et al. [55]	1.4916 ± 0.0003	$\nu = 2.25 \pm 0.01$	0.0200 ± 0.0005	0.001 ± 0.001

Figure 2: Critical exponents, MFA: mean field approximation, PT: perturbation theory, MC: Monte Carlo Simulation, HTSE: high temperature series expansion

This is the so called *dimensional reduction* (c.f. figure 2, row 3). If we compare this with

$$\nu(d - \theta) = 2 - \alpha \tag{28}$$

this would yield $\theta = 2$ in all dimensions. This leads to the conclusion that since the lower critical dimension of the pure Ising model is $d_l = 1$ the RFIM should have $d_l = 1 + 2 = 3$ in disagreement with what we found before (the 3d RFIM actually has been proven to exhibit LRO). For quite a while this was a puzzling result and many papers concluded that there must be something wrong with the Imry-Ma argument. They constructed some bizzare modifications in order to obtain $d_l = 3$. Only in recents years have physicists realized that the mistake was at the other end, namely that perturbation theory is not appropriate to deal with this type of disorder and that dimensional reduction is wrong. The breakdown of perturbation theory is mainly due to a large number of local minima in the energy landscape. Again we see that the RFIM has quite surprising features.

8 Dilute Antiferromagnets

In the last sections we saw a lot of theoretical considerations and predictions. We saw that in the presence of weak random fields LRO is destroyed in dimensions $d \leq 2$ and that it has been rigorously proven that LRO exists in $d = 3$. Nevertheless, it is not yet clear what the nature of the phase transition is. The interesting case is obviously the one where LRO first occurs, i.e. $d = 3$ and fortunately this is the dimensionality where experiments can most easily be performed. We would like to measure critical exponents and compare the results with those obtained by simulations and calculations. Unfortunately, even though the RFIM was introduced more than 20 years ago, it remained outside the realm of experimental study for many years. No mechanism was known for easily generating fields in a controlled way that coupled to the order parameter with uncorrelated randomness on the scale of the crystal lattice spacing. This changed in 1979 when Fishman and Aharony [16] proposed a physical realisation of such a mechanism. They showed that an external uniform field applied to an anisotropic antiferromagnet with random exchange coupling will generate random fields coupling to the antiferromagnetic order parameter (staggered magnetisation). The Hamiltonian of this system is given by

$$H = \sum_{<ij>} J_{ij} \epsilon_i \epsilon_j S_i S_j - H \sum_i \epsilon_i S_i \tag{29}$$

where $\epsilon_i = 1$ if the site is occupied and 0 if it is empty. H is the uniform applied field. The local random fields in this system are generated by competition between the alignment of the local sublattice that has the most spins with the applied field and the long range antiferromagnetic order where one sublattice globaly alignes with the field. The applied uniform field and the effective random field are proportional to each other. The random field can therefore easily be controled and even switched off completely. This provides the opportunity to do scaling studies in a systematic way and also allows for so called zero-field-cooling (ZFC). This means nothing more than cooling down the sample without any random fields present. For some systems other than the dilute antiferromagnets (DA), one has no control over the strength of the random fields and it is therefore only possible to perform field cooling (FC), i.e. cooling the sample in presence of random fields. Since hysteresis plays an important role in interpreting the data for the RFIM ZFC is crucial.
The important point, however, that makes the diluted anitferromagnets (DA) so useful for studying the RFIM is, that both systems belong to the same static critical behaviour universality class. Otherwise, critical exponents obtained by measuring DA would not yield any useful information about the critical behaviour of the RFIM. The progress in understanding the RFIM is strongly coupled to experiments on high-quality DA crystals.

9 Crossover Phenomena

Crossover phenomena are an other interesting and important subject one has to be aware of when performing experiments on RF systems. This relates to interpretation of experimental data and the comparability with theoretical predictions. By crossover phenomena we mean nothing more than that at some larger value of $|t|$ one observes behavior resembling a certain universality class. Approaching the critical temperature more closely one might observe the crossover to critical behaviour belonging to another universality class with different exponents. In our case this would be the crossover from non-random to random field behaviour as

T_c is approached in weak random fields. This fact is integral in understanding the random field problem. The free energy in presence of random fields scales as

$$F = |t|^{2-\alpha} f(t h_r^{-2/\phi}) \qquad (30)$$

which is valid for $h, t \to 0$ and t is given by $t = (T - T_c(0) + bh_r^2)/T_c(0)$. Here $T_c(0)$ is the transition temperature in zero field, h_r is the effective random field (c.f. equation 5) and b a mean field correction due to the random fields. The exponent ϕ is the so called *crossover exponent.* For the RFIM one finds $\phi = \gamma$, where γ is the susceptibility exponent of the pure system. For the diluted antiferromagnets ϕ is somewhat larger as γ with $\phi = 1.42 \pm 0.03$ for the system $Fe_x Zn_{1-x} F_2$ as an example. Another consequence of the crossover function for the DA is a depression in the phase transition temperature, given by

$$t_c(H) = -cT_c(0)h_r^{2/\phi} \quad (d = 3) \qquad (31)$$

where c is a non universal constant. A general statement is that for weak random fields all relevant reduced temperatures scale with h_r in the combination $t h_r^{-2/\phi}$. One can show that the crossover from non to RFIM behaviour will occur for

$$|t| < t_{cross}(t) = a h_r^{2/\phi} \qquad (32)$$

where a is a non universal constant. This means that far from $T_c(0)$ and with small random fields the system shows critical behaviour not characteristic of the RFIM. Only when $T_c(0)$ is closely approached one can observe the crossover to random field behaviour. This means that if one performs measurements on some experimental realisation of the RFIM one has to make sure to go beyond t_{cross} in order to measure the correct critical behaviour.

10 Experimental Results

In this last section we will look at experimental results, mainly specific heat measurements, and will check if they confirm the theoretical predictions obtained in the preceeding sections. We first turn to the case $d=2$. We expect no transition to a long range ordered state if we cool down the sample . For weak random fields the system will appear to approach the transition with exponents of zero random fields, i.e. it will seem to exhibit LRO. For $|t| < t_{cross}$ the behaviour will change and only a rounded maximum in the specific heat will occur, since the free energy does not exhibit a singularity. With small enough random fields the transition will in practice be too small to be observed. The behaviour of the specific heat is shown in figure 3. The data is obtained by using an optical technique. In DA $\frac{d\Delta n}{dT}$ is proportional to the

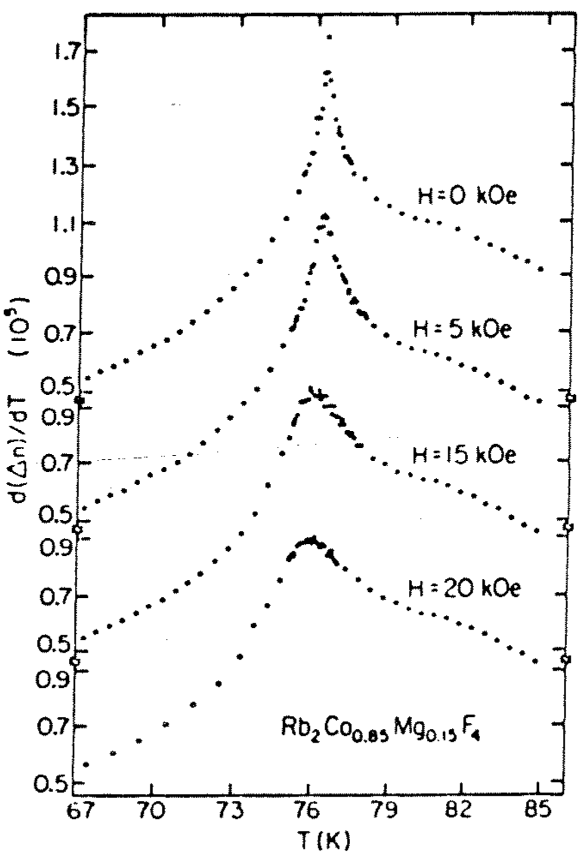

Figure 3: specific heat for $2d$-system $Rb_2Co_{0.85}Mg_{0.15}F_4$

magnetic specific heat. The material used is layered magnetic $Rb_2Co_xMg_{1-x}F_4$ with $x = 0.85$, which constitutes an extremely good $2d$ Ising system. The data shows very clearly that for zero field a sharp (to the extent of experimental resolution) transition occurs, whereas for small random fields the transition is rounded. This corressponds to the destruction of LRO. The divergence observed in zero field is (symmetric) logarithmic. One can extract the crossover exponent out of the data and obtains $\phi = 1.58 \pm 0.22$. This agrees reasonably well with the staggered susceptibility exponent $\gamma = 1.75$. Summarizing it can be stated that for $d=2$ specific heat experiments are consistent with theoretical predictions and computer simulations and show the destruction of LRO. People also did neutron scattering experiments. I will not go into details, but only state that they also agree with theory and simulations.

Now we turn to the case $d=3$ where we expect LRO according to the considerations in the preceeding chapters. This case, however, turns out to be much more complex and difficult to study than the $d=2$ case. This is because dynamical effects play a major role. There are other complications that arise in studying DA. For example the presence of magnetic impurities in DA constitute strong pinning sites for the domain walls and this has to be taken into account if one wants to compare experimental and theoretical data, since this feature is not present in theoretical studies of the ferromagnetic model. Another important point are concentration gradients. In preparing the crystals one has to be very careful in order not to introduce any variation in the magnetic dilution. Otherwise concentration gradients arise. Since the transition temperature strongly depends on the concentration the sample would then have a locally varying transition temperature. This would strongly affect the measurable critical behaviour. Typically, concentration gradients have to be small enough such that the variation of $T_c(H)$ over the whole sample is of the order of 10^{-3} of $T_c(H)$ or less if one wants to observe RFIM critical behaviour. In addition concentration gradients tend to round the transition making it hard to predict accurate values for the exponents. In general, gradients have to be determined independently and have to be incorporated in the interpretation of the data. We allready mentioned crossover phenomena in the preceeding section. When taking data one has to make sure to be close enough to T_c in order to measure the correct universality class behaviour.

The most studied DA system in $d=3$ is $Fe_xZn_{1-x}F_2$ for arbitrary $0<x<1$. Critical behaviour for this system has been observed for $|t|<0.02$. We again focus on specific heat results obtained by an optical technique. An advantage of this technique with regards to the complications mentioned above, is that the effects of concentration gradients can be minimised by orienting the laser beam perpendicular to the concentration gradient. The results are shown in figure 4 for $Fe_{0.47}Zn_{0.53}F_2$. The magnetic field (the sample is ZFC in this experiment) is increasing from (a) to (d). The right arrow indicates the crossover from non to RFIM behaviour, the left one indicates the onset of rounding due to dynamical effects. The slow dynamics of the RFIM near T_c is due to an exponentially diverging relaxation time $\tau \propto e^{c/\xi^\theta}$. If the time of the measurement is less than the relaxation time the system drops out of equilibrium. One can see that the ZFC peaks exhibit symmetric logarithmic behaviour over a range of $|t|$ that increases with the applied field. The logarithmic divergence in the RF case implies $\alpha = 0$. Figure 5 shows recent data (from [9]) for the specific heat, taken for $Fe_{0.93}Zn_{0.07}F_2$. In the right part of figure 5 the phonon contribution has

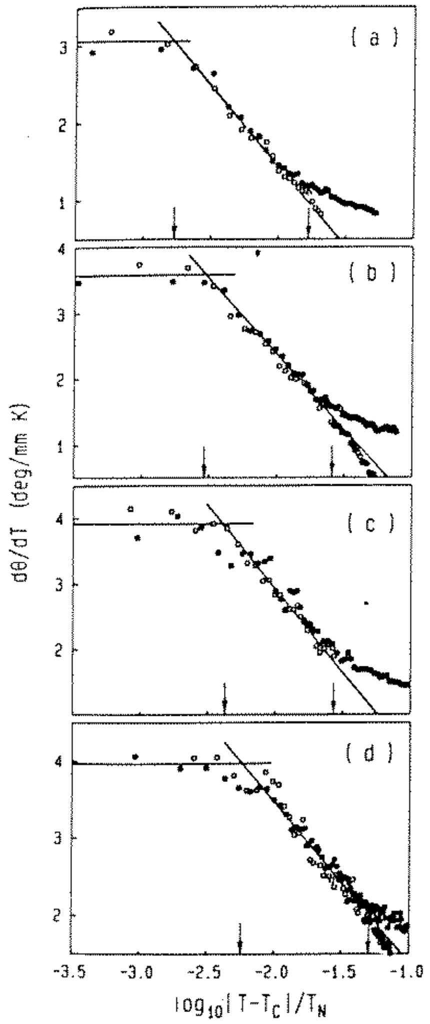

Figure 4: specific heat for $Fe_{0.47}Zn_{0.53}F_2$

already been subtracted. The inset shows the FC data that is much more rounded due to frozen in finite size metastable clusters. In conclusion, it can be said that high resolution specific heat measurements suggest the existence of a logarithmic divergence with no evidence of any accompanying background discontinuity in $d=3$. From this the expected LRO in three dimensions can be infered. Recent Monte Carlo simulations, unfortunately, do indicate a cusp with a large negative exponent. This discrepancy between experiment and simulation is yet unsolved.
People also did neutron scattering to measure the scattering amplitude. Unfortunately there does not exist a good theoretical prediction for the line shape in the cases $t<0$ and $t>0$ and it is therefore difficult to interpret the data and to extract the critical exponents. I will not go into this difficult subject. Fits to the experimental scattering data suggest the following values for the critical exponents:

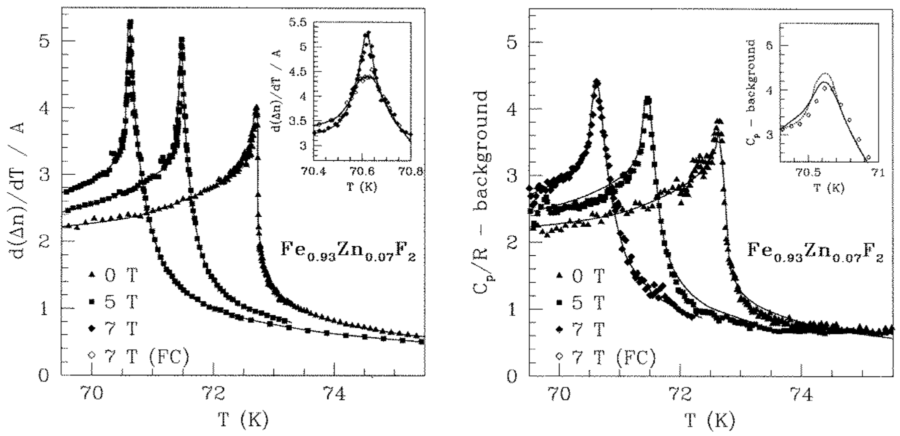

Figure 5: specific heat for $Fe_{0.93}Zn_{0.07}F_2$

$\nu = 0.93 \pm 0.03$, $\gamma = 1.71 \pm 0.06$, $\bar{\gamma} = 3.0 \pm 0.1$ for $10^{-3} < t < 10^{-2}$. Those are in reasonable good agreement with results obtained by Monte Carlo simulations that yield $\nu = 1.1 \pm 0.2$, $\gamma = 1.7 \pm 0.2$, $\bar{\gamma} = 3.3 \pm 0.6$.

11 Conclusion

The purpose of this paper was to point out the effect of random fields on ideal systems. It was shown that many features of the ideal models change, e.g. the lower critical dimension, the critical exponents, the scaling laws and the character of the phase transition in the case of the RFIM. The RFIM is a field of intensive current research. There are yet many discrepancies between experiments and the predictions made by theory and simulation. It should be realized that one has to be careful in applying models to physical reality especially if one wants to compare experimental data with theoretical predictions. There might be subtleties involved, such as crossover phenomena for example. The case of the RFIM model shows that even arbitrary weak deviations from ideal behaviour lead to drastic changes in the physics.
I hopefully did not confuse the reader too much but encouraged him to think more about this interesting subject.

References

[1] Y. Imry and S.K. Ma, *Phys. Rev. Lett.* **35**, 1399 (1975)

[2] T. Nattermann and J. Villain, *Phase Transitions* **11**, 5 (1988)

[3] D.P. Belanger, *Phase Transition* **11**, 53 (1988)

[4] D.P. Belanger and A.P. Young, *Journ.of Magn.and Magn.Mat.* **100** (1991)

[5] T. Nattermann, cond-mat/9705295

[6] J.-C. Angles d'Auriac, cond-mat/9704088

[7] H. Rieger, cond-mat/9503041

[8] H. Rieger and A.P. Young, cond-mat/9305045

[9] D.P. Belanger, cond-mat/9706042

[10] K. Binder, *Z. Phys B* **50**, 343 (1983)

[11] A.J. Bray and M.A. Moore, *J. Phys. C. Solid State Phys.* **18**, L927 - L933 (1985)

[12] R.B. Stinchcombe and E.D. Moore and S.L.A. de Queiroz, cond-mat/9506056

[13] A. Aharony and Y. Imry and S. Ma, *Phys. Rev. Lett.* **37**, 1364 (1976)

[14] J. Imbrie, *Phys. Rev. Lett.* **53**, 1747 (1984)

[15] Y. Imry, *J. of Stat. Phys* **34**, 849 (1984)

[16] S. Fishman and A. Aharony *J. Phys C* **12**, L729 (1997)

[17] M. Schwartz and A. Soffer, *Phys. Rev.* **33**, 2069 (1986)