Metal-Insulator Transition (Scaling Theory)

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Abstract

In this paper I briefly discuss the concepts of localization and metal-insulator transition. The phenomenological description of the conducting properties depending on the dimensionality and disorder, and the physical arguments for the scaling β-function are presented. The conducting properties of the 3D system near the critical point of the metal-insulator transition predicted by this scaling model are also discussed. It is shown how the corresponding critical exponent can be calculated from the $2 + \epsilon$ expansion. I also give the formulas for the weak-localization corrections to the conductivity for the one-, two- and three-dimensional systems and their dependencies on frequency and temperature. The limits of validity of this theory and ways of its confirmation from the microscopic point of view are also considered.

I. Introduction

It is well known that the state of an electron in a periodic crystal is described by the Bloch wavefunctions. However, in reality the solids are not perfectly ordered; some amount of disorder is always present. When the concentration of imperfections is small enough, it is customary to use the approximation that describes the scattering of Bloch waves by random impurities. From this one can derive the Boltzmann transport equation for the conductivity. But what happens if the disorder becomes stronger? In this case the conventional description using the Bloch functions appears to be totally wrong. Strictly speaking it is wrong even in the limit of weak disorder, because in the presence of imperfections the effective crystalline potential is no longer periodic, and we encounter the problem of finding the eigenstates of an electron moving in the random potential [1]. In the pioneering paper Anderson showed that the electron wavefunction can be profoundly changed if the disorder is sufficiently strong [2]. The conventional Bloch wavefunction is extended throughout the sample though in the presence of the weak randomness its phase coherence is destroyed at distances of the mean free path l. Anderson showed, that when the disorder is strong enough, a wavefunction may become localized, i.e. it decays exponentially from some point in space:

$$|\psi(\mathbf{r})| = \exp(|\mathbf{r} - \mathbf{r}_0|/\xi) \tag{1}$$

The parameter ξ is called the localization length.

It appears that the character of localization is strongly dependent on dimensionality. It can be rigorously shown that in 1D all states are already localized in the arbitrary weak disorder. The existence of extended states in 2D is the question that is not completely resolved at present. The picture is more transparent in the 3D case. Definitely in the limit of strong disorder all states are localized. But what happens in case of the intermediate disorder? Anderson considered the simple model Hamiltonian for the impurity band with one orbital [1]:

$$H = \sum_n \epsilon_n a_n^\dagger a_n + V \sum_{n,m} a_n^\dagger a_m \tag{2}$$

Here ϵ_n are the single site energies that are assigned at random, and V is the hopping matrix element between nearest-neighbour sites. This model allows us to understand how the localization-delocalization transition can occur [4]. Let's assume that the site energies are distributed uniformly over interval W. When $W = 0$, we have that all sites have the same energy that corresponds to the ordered crystal and unscattered Bloch states. But when $V = 0$, there is no connection between sites and consequently there is no transport at all. This means that in the limit $V = 0$ all states are localized. From the other side when W is rather large and

V is small, we have many localized states with energies distributed over the broad interval. Because of finite V these states are mixed. Their admixture, however, cannot produce any extended states, because V is small and differencies in energies entering the denominators in the terms of the perturbative series for the wavefunction are large. So we come to the conclusion that in 3D there is some critical value of $(W/V)_c$, above which the existence of extended states is impossible. When $(W/V) < (W/V)_c$, we have both extended and localized states that are separated in energy from each other. For $E < E_c$ the eigenstates are extended, and for $E > E_c$ they are localized. The boundary energy E_c is called the mobility edge.

II. The Scaling Theory of Localization and Metal-Insulator Transition

In order to solve the equation (2) at least approximately one should use the Green function technique. However, as was shown by Anderson, though this method can predict quantitatively the localization transition, the dimensional aspect of localization is supressed in it. The dimensional dependence of the metal-insulator transition can be accounted by the scaling theory of localization that was first introduced in the works of Thouless [5] and later developed by Abrahams, et al.[6]. Below we'll discuss the conceptual framework of this theory.

Suppose we have 2^d cubic samples of size L^d in d dimensions and put them together to form one cubic sample of size $(2L)^d$. Let's ask ourselves about the nature of the eigenstates in this new sample. Definitely, the eigenstates of the $(2L)^d$ sample will be similar to those of the L^d sample. It's natural to assume that the eigenstates of the larger sample will be some linear combinations of the eigenstates of the smaller sample. The quantity of interest is the ratio $\Delta E/\delta W$. Here δW is the typical spacing between the energy levels in the L^d sample and can be represented by the formula $\delta W = (\rho L)^{-1}$ with ρ being the density of states. ΔE is the overlap integral between the states. It can be calculated if we assume, for example, the periodic boundary conditions. If we make the sample repeated in space, each energy level of it will broaden out to form a band, and the bandwidth will have the same order of magnitude as the overlap integral. If the eigenstates are localized, ΔE will be insensitive to the change in the size of the sample, because the effect of other eigenstates located far apart will be exponentially small. From the other side ΔE is large for the extended state, because such a state becomes extended over the large new region of space. So, the ratio $\Delta E/\delta W$ seems to be the only parameter that reflects the localization properties in a sample of size L^d. This ratio that will be denoted as G is some function of L. Then $G(L) = \Delta E/\delta W = \rho L^d \Delta E$. This quantity G, the so-called conductance, is a physically measurable quantity. It relates the eigenstates of a sample of size $(2L)^d$ to those of an original sample of size L^d. Usually one uses the dimensionless conductance $g = G/(e^2/\hbar)$, and it can be considered as the single parameter that controls the localization behaviour of the system as a function of size L.

Let's determine the limiting behaviour of the conductance in the limit of weak disorder. In this case, as was noticed by Thouless, $\Delta E = \hbar/\tau$, where τ is the time of diffusion from the center of a sample to its edge, i.e. $\tau = L^2/D$. Using the Einstein relationship for the conductivity $\sigma = e^2\rho D$ with D being the diffusion coefficient, we get the result that

$$g(L) = G(L)/(e^2/\hbar) = \sigma L^{d-2} \tag{3}$$

This formula is simply the Ohm's law for a sample. Deriving this we assumed the diffusive motion of electrons. This is true, if the system is large enough to have the well-defined mean free path l, i.e. $L \gg l$. In this case σ is an intensive quantity independent of L.

If the states near the Fermi energy ε_F are localized, then the hopping matrix element between them is exponentially small, because the states being close in energies are very far apart in space. The relevant length in this case is the localization length ξ, that is generally greater than the mean free path l. For $L \gg \xi$, one expects the non-Ohmic dependence of the conductance:

$$g(L) \propto \exp(-L/\xi) \tag{4}$$

Note that both (3) and (4) describe the large L behaviour of conductance. For the given disorder $g(L)$ evolves smoothly to (3) or (4) with increasing L. The limiting behaviour depends on microscopic disorder, i.e. on the value g_0 at scale l, and on the dimensionality of the system. So the main goal of the phenomenological scaling theory is to describe how $g(L)$ changes with L, provided that we know the value g_0.

The key idea of Abrahams, et.al is that the β-function that describes the scaling trajectory, being the logarithmic derivative of g with respect to L, is dependent only on one parameter g:

$$\frac{d\ln g(L)}{d\ln L} = \beta(g) \tag{5}$$

The reason for this is that the change in the effective disorder with the change in the scale of the system is determined only by the disorder at the previous lengthscale, i.e. by the conductance that serves as the only parameter characterising disorder.

Let's consider now what forms of $\beta(g)$ should be expected in different limiting regimes. In case of the large conductance, when formula (3) is valid, we get by taking the logarithmic derivative:

$$\beta(g) = (d-2) \tag{3a}$$

In two dimensions $\beta(g)$ tends to zero. This is the consequence of the fact that in 2D g and σ have the same physical dimension.

In the case of small conductance, when we should use formula (4), we get

$$\beta(g) = \ln \frac{g}{g_c} \tag{4a}$$

Here g_c is the characteristic intermediate dimensionless conductance. The expression (3a) is valid in the limit $g \gg g_c$, while (4a) holds for the opposite case $g \ll g_c$. g_c must be of the order of unity. It appears to be equal to $(\pi)^{-2}$ for the spin-1/2 noninteracting electrons.

Abrahams, et.al suggested that the righthand side of (3a) describing the regime of extremely weak disorder is in fact only the first term of the expansion on the small parameter $1/g$. In terms of the mean free path l and the Fermi wave vector k_F the weak disorder can be characterised as the limit when $k_F l \gg 1$. Gor'kov, et.al showed using the perturbation theory on $1/k_F l$ that there are significant corrections to the conductivity if we take into account multiple backscattering processes. These processes give the contribution to the β-function proportional to $1/g$:

$$\beta(g) = (d-2) - a/g \tag{5}$$

This correction is negative reflecting the fact that the backscattering diminishes the conductivity. So the conductivity of a weakly disordered sample is never perfectly Ohmic. The constant a can be calculated analytically for the system of noninteracting spin-1/2 particles and was found to be equal to (π^{-2}). The scaling curve that corresponds to the large g behaviour described by equation (5) is shown at the Figure. While constructing this figure it is reasonable to assume that $\beta(g)$ is continuous and monotonic. The former assumption stems from the fact that the system evolves continuously with increasing disorder to the localized behaviour. The latter assumption admits exceptions [1]. These situations are exotic and will not be considered here. Let's discuss the consequencies of the scaling equation (5) for different dimensions. In 3D for large g $\beta(g)$ tends to unity. When g gets smaller $\beta(g)$ decreases and at small enough g the beta-function is negative. So there will be some critical value of g_3 at which $\beta(g_3) = 0$. The quantity that we are supposed to know initially in this scaling theory is the conductance g_0 at the lengthscale l. Suppose g_0 is greater than g_3. In this case we start from the point where $\beta(g)$ is positive, and the slight increase of L will result in the increase of g. So, increasing further the lengthscale we move up on the curve $\beta(g)$ until we reach the almost Ohmic behaviour at very large L. From the other side if at the lengthscale l $g_0 < g_3$, we start from the negative side of the scaling curve, and any increase of L will result in the decrease of g. So, we move down on the β-curve, which means that we reach at large enough L the situtaion where all states are localized. Thus, we see how the conducting properties of the material depend only on one parameter g_0: if $g_0 > g_3$, the system is a metal, if $g_0 < g_3$ the system is an insulator. Using the terminology of a fixed point one can say that $g = g_3$ is an unstable fixed point, because small deviations take us further away from this point leading eventually to two qualitatively different regimes, depending on whether the flow started below or above g_3.

We know that g_3 is determined by the degree of disorder at the given concentration of electrons. But we can also keep the disorder in our system constant and vary the concentration of electrons n that is connected with the Fermi energy ε_F. The fixed disorder means the fixed mobility edge E_c. Because the transport properties of the system of electrons are determined by the states in the vicinity of the Fermi surface, g_0 will be a function of ε_F at the lengthscale l. As can be understood from the introduction, if $\varepsilon_F > E_c$, the states near the Fermi energy are localized, and the system is insulating. So, we can say that the critical point of the metal-insulator transition is when the Fermi energy is equal to the mobility edge, i.e. $g_3 = g_0(E_c)$. When the Fermi energy is close to the mobility edge, one can expand the function $g_0(\varepsilon_F)$:

$$g_0(\varepsilon_F) = g_3 + (\varepsilon_F - E_c)g_0' \tag{6}$$

with $g_0' = (dg_0/d\varepsilon_F)$ at $\varepsilon_F = E_c$. So, the behaviour near the mobility edge is determined by the predictions of the scaling theory for g_0 close to g_3. The conductivity can be considered as a function of closeness of the Fermi energy to the mobility edge. The corresponding critical exponent with which the conductivity turns to zero at $T = 0$ when $\varepsilon_F \to E_c$ is denoted μ, so that $\sigma(T = 0) = (\varepsilon_F - E_c)^\mu$.

We know that at the fixed point $\beta(g) = 0$. Suppose that near g_3 this function has the slope $1/\nu$. First we determine the relation of ν to the character of conductivity near the mobility edge and then show how this exponent can be determined from the ϵ-expansion. For small $\delta g = (g - g_3)/g_3$ we have

$$\beta(g) = \frac{1}{\nu}\left(\frac{g - g_3}{g_3}\right) = \frac{\delta g}{\nu} \tag{7}$$

Consider first the case $\delta g > 0$. Integrating (5) describing the behaviour at large lengthscales L and equation (7) which is valid at small lengthscales l, and matcing then the solutions at some intermediate scale, we get that $g(L) = \sigma L$ with

$$\sigma = (Ag_3/l)(\delta g)^\nu \tag{8}$$

Here A is some constant of the order of unity that appeared as a result of integration. We see that while approaching the mobility edge the conductivity scales to zero with the universal critical exponent ν. Rewriting the equation (8) in the form $\sigma = g_3/\xi$, we see that the correlation length ξ being the only relevant length parameter near the point of transition is given by

$$\xi = \left(\frac{l}{A}\right)(\delta g)^{-\nu} \tag{8a}$$

and diverges with the conductivity exponent while approaching the transition point. This result can be also interpreted in the way that the conductivity has clearly non-Ohmic character at lengthscales smaller than ξ and almost Ohmic at lengthscales much greater than ξ. This justifies the definition of ξ as the correlation length analogous to other critical phenomena.

When $\delta g < 0$, the integration of equations (7) and (5) as was done above gives

$$g(L) = g_3 \exp(-B|\delta g|^{\nu} L/l) \tag{9}$$

with B being a constant of the order of unity. Thus our system is in the localized regime with the localization length

$$\xi_{loc} = (l/B)(\delta g)^{-\nu} \tag{9a}$$

The localization length goes to infinity on approaching the mobility edge. We also observe that the localization length on the insulating side and the correlation length on the metallic side tend to infinity with the same critical exponent ν, while the prefactors are different on both sides and depend on the details of the system.

We now turn to the question of calculation the universal exponent ν using the perturbative ϵ-expansion. From equation (5) it can be seen that the existence of the critical point is determined by the dimensionality of the system. $\beta(g)$ can turn into zero at some point only if $d > 2$. So it makes sense to consider the calculation of the critical exponent ν in the imaginary system of $2 + \epsilon$ dimension and then extrapolate the result to $\epsilon = 1$. Suppose first that $\epsilon \ll 1$. In this case equation (5) reads

$$\beta(g) = \epsilon - a/g \tag{10}$$

The critical point is determined by

$$g_{2+\epsilon} = (a/\epsilon) \tag{10a}$$

This value is large for small ϵ. Due to this the perturbative expansion (10) is well-controlled near the critical point, because other so far neglected terms in the expansion of the β-function will have the additional order of smallness in the small parameter $1/g_{2+\epsilon}$. The slope of the curve $\beta(g)$ ν^{-1} is equal to ϵ. Performing the integration of (10) directly we come to the result that

$$\sigma = (A g_{2+\epsilon}/l^{\epsilon})(\delta g)^{\mu} \tag{11}$$

To the lowest order in ϵ we find $\mu = \nu\epsilon = 1$ and $\nu = 1/\epsilon$. The critical conductivity will be of the form

$$\sigma = A g_{2+\epsilon}/\xi^{\epsilon} \tag{12}$$

with the correlation length ξ diverging as

$$\xi = l(\delta g)^{-\nu} \tag{12a}$$

Extrapolating this result to $\epsilon = 1$ one gets simply $\nu = 1$. Though this result is approximate, we see that the calculation of the critical exponent by the method of ϵ-expansion is very simple in the case of one parameter scaling.

Let's discuss now what will be observed in case of a strictly two-dimensional system. In this case the β-function is always negative. This means that at large enough lengthscales the system can be only in the localized state. Suppose the conductance g_0 is large at l. Then upon rescaling we'll find ourselves down in the β-curve. This means that there is no truly Ohmic behaviour in two dimensions. Integrating equation (5) in 2D from the initial value g_0 at some lengthscale l we get the result for conductance ($a = e^2/(\pi^2\hbar)$ in the usual system):

$$g_{2D}(L) = g_0 - \frac{e^2}{\hbar\pi^2}\ln\frac{L}{l} \tag{13a}$$

Using the simple Boltzmann transport equation one gets in 2D that $g_0 = (e^2/2\pi\hbar)(k_F l)$. So the localization correction to the conductivity becomes comparable to the regular diffusion conductivity when the size of the system is comparable to the localization length that is given by

$$\xi_{loc} \simeq l\exp(\hbar\pi^2 g_0/e^2) = l\exp(\frac{\pi}{2}k_F l) \tag{14}$$

The localization length is exponentially large at weak disorder that is determined by the condition $k_F l \gg 1$. This means that though the conductivity is non-Ohmic at all lengthscales the weak localization correction is difficult to observe in real samples with weak disorder.

Finally, we consider what should be expected from the scaling theory in 1D. The perturbative expansion of the β-function used by us previously leads to the result that in strictly one dimension the localization correction becomes comparable to the Boltzmann conductivity at some lengthscale of the order of l. This means that all the states are deeply localized at any disorder and the resistance of a given sample varies exponentially with its length. One should make here the following remark, that belongs to Anderson, et.al [8]. The perturbative expansion (5) is given for the conductance, averaged over all possible realizations of disorder. In 1D systems the fluctuations of the average conductance are so strong that the actual resistivity can in fact have the value very different from the average. As was emphasized in [8] g is not well-behaved quantity in this sense, i.e. it does not tend to some typical value averaged over disorder, when L goes to infinity. In other words, the operations of taking the limit $L \to \infty$ and the averaging over disorder do not commute for g. The distribution is well-behaved, however, for the parameter $\ln(1 + 1/g)$ that can be taken as a scaling variable. The average of this parameter has the additive property. Thus one can write $\ln(1 + 1/g) = \alpha L$, where α has the natural meaning of the inverse localization length. Rewriting $g^{-1} = \exp(\alpha L - 1)$ one can calculate the β-function:

$$\beta(g) = -(1+g)\ln(1+g^{-1}) \tag{15}$$

Being expanded at large g this expression gives:

$$\beta(g) = -1 - \frac{1}{2g} + \frac{1}{6g^2} \tag{15a}$$

Notice, that the coefficient before $1/g$ in this expansion $-1/2$ is different from that in the formula (5) calculated from the perturbative diagrammatic technique.

Here one should make the following comment. The above form of the scaling function is strictly speaking valid only for the system of non-interacting electrons. This was proven rigorously in [6] and [7] by the method of diagrammatic technique for the system of electrons interacting with random impurities in the limit $k_F l \gg 1$. The formulas analogous to (13a) for 2D conductivity of a sample of the size L were derived also for the three and two dimensional systems and are given by

$$\sigma_{3D} = \sigma_0 - \frac{e^2}{\hbar\pi^3}\left(\frac{1}{l} - \frac{1}{L}\right) \tag{13b}$$

$$\sigma_{1D} = \sigma_0 - \frac{e^2}{\hbar\pi}(L-l) \tag{13c}$$

$\sigma_0 = ne^2\tau/m$ is the usual Drude conductivity. These formulas lead to the equation (5) for the β-function if one defines the conductance as $g = \sigma(L\pi)^{d-2}$.

Returning to the formula (13a) we notice that it was derived by the integration of equation (5) in two dimensions. By assumption the β-function does not depend explicitly on L. In order for this to hold, the next term in $(k_F l)^{-1}$ in (13a) must vanish, i.e. in the expansion of the conductivity

$$\sigma_{2D}(L) = \sigma_0\left(1 - c_1\frac{1}{k_F l}\ln\frac{L}{l} + c_2\frac{1}{(k_F l)^2}\ln^2\frac{L}{l} + ...\right)$$

the coefficient c_2 must vanish. This is really the case as was shown in [7] again for the case of non-interacting electrons. This circumstance gives the strong support for this beautiful semi-phenomenological scaling theory.

So far we considered only the static conductivity. The dynamic behaviour of the conductivity is also very interesting, because now we have the new characteristic scale in the time dimension, i.e. the inverse frequency ω^{-1}. As was shown in [7] by means of the diagrammatic technique for disordered systems, ω can play the same role of a cutoff as the sample size L.

The corresponding dimensionless parameter in this case is $\omega\tau$, where $\tau = l/v_F$ is the relaxation time. So the frequency- dependent conductivity in 2D is given by:

$$\sigma = \sigma_0 \left[1 + \frac{1}{k_F l} \ln(|\omega\tau|)\right] \tag{16a}$$

and in 3D by

$$\sigma = \sigma_0 \left[1 + \frac{3\sqrt{6}}{4(k_F l)^2}\sqrt{\omega\tau}\right] \tag{16b}$$

Here we assumed that $\omega\tau \ll 1$ and set σ_0 independent of ω. These formulas suggest the unusual non-Drude frequency dependence of the conductivity and are derived for the case of zero temperature. The localization correction in 3D is not interesting due to the large parameter $(k_F l)^2$ in the denominator and small $\omega\tau$ in the numerator. In 2D the conductivity has the logarithmic dependence on frequency. The characteristic frequency at which one observes the crossover from the Drude-like to the localized behaviour is $\omega_0 \sim (1/\tau)\exp(-\pi k_F l)$. ($\omega_0\tau \ll 1$ when $k_F l \gg 1$). Notice the formal analogy between (16a) and (13a). The characteristic time of diffusion at the lengthscale L is $\tau_L \sim L^2/D(L)$ and plays in the static problem the same role of cutoff as ω^{-1} in the dynamic case. If $D(L)$ depends weakly on L then, $\sqrt{\omega}$ scales as L^{-1} and $\ln(1/\sqrt{\omega\tau}) \propto \ln(L/l)$.

Near the metal-insulator transition the diffusion constant $D(L) = (\sigma/\rho)(e^2/\hbar)^{-1}$ and varies with length as

$$D(L) = \frac{g^*}{\rho L^{d-2}} \tag{17a}$$

where g^* is the typical value of the conductance at the fixed point. The connection between ω and L is now $\omega \approx D(L)L^{-2} = g^*/(\rho L^d)$. From this we see that the new characteristic lengthscale is given by

$$L_\omega = \left(\frac{\omega}{\rho}\right)^{-1/d} \tag{17b}$$

Putting this lengthscale into (17a) and taking into account that $D(L) \propto \sigma$, we get

$$\sigma \propto \omega^{(d-2)/d} \tag{18}$$

Thus near the point of the metal-insulator transition in 3D the conductivity goes as $\omega^{1/3}$.

The most general frequency dependent scaling form was suggested by Wegner [9] and has the form

$$\sigma(\omega) = \frac{e^2}{\hbar\pi^2}\xi^{2-d} f(\xi/L_\omega) \tag{19}$$

For $L_\omega \gg \xi$ corresponding to the case of low frequencies we get the ω-independent result $\sigma(\omega) \sim (e^2/\hbar\pi^2)\xi^{2-d}$ derived earlier. For this one must have $f(x) \to 1$ as $x \to 0$. In the opposite limit $L_\omega \ll \xi$ corresponding to the case of large frequencies $\sigma(\omega)$ must be independent of ξ. So, for $x \gg 1$ one must have $f(x) \sim x^{d-2}$.

The scaling results presented so far are valid for the zero temperature. However, they can be generalized to the finite temperatures in the following way. At non-zero temperatures electrons always experience the inelastic scattering. As was argued by Thouless [10] the inelasting scattering introduces random fluctuations in the time evolution of an electronic state. Such fluctuations destroy the quantum coherence necessary for localization. Let the characteristic relaxation time for the inelastic scattering is τ_{in}. During this time the electron will diffuse the distance $L_{in} = \sqrt{D\tau_{in}}$. $D = v_F^2\tau/d$ is the usual diffusion constant in d dimensions. At the zero temperature the size of a sample L served as a natural cutoff while calculating the static conductivity. At nonzero temperatures the role of the cutoff is played by L_{in}. τ_{in} increases with decreasing temperature. If the temperature is not too small, then $L_{in} \ll L$ for samples of the macroscopic size. Suppose that $\tau_{in} = \alpha T^{-p}$ where $p > 0$ and depends on the scattering mechanism and, generally speaking, the dimensionality of the system. Then $L_{in} = \alpha T^{-p/2}$. The corresponding formulas for the conductivity with this cutoff will take the form

$$\sigma_{2D}(T) = \sigma_0 + \frac{p}{2}\frac{e^2}{\hbar\pi^2}\ln\left(\frac{T}{T_0}\right) \tag{20a}$$

$$\sigma_{3D}(T) = \sigma_0 + \frac{e^2}{\hbar\pi^3}\frac{T^{p/2}}{\alpha} \tag{20b}$$

$$\sigma_{1D}(T) = \sigma_0 - \frac{\alpha e^2}{\hbar\pi}T^{-p/2} \tag{20c}$$

Looking at these dependences we see that the conductivity decreases with decreasing temperature driving the system into the localized regime. However, the functional dependence of this decrease is different for different dimensions.

III. Conclusion

In conclusion we make some comments about the validity of the presented scaling theory of localization and metal-insulator transition. It was several time emphasized throughout the paper that this theory can be proved by the corresponding microscopic considerations only for the non-interacting electrons. What may happen to the system of interacting electrons? If the interaction is weak, one can treat it as a perturbation. The perturbative account of the interaction does not change qualitatively the above results leading simply to the innocuous renormalization of constants. But in real systems the interaction is usually strong. Moreover, very often we encounter the situation when not only the interactions but also the disorder are strong, i.e. the condition $k_F l \gg 1$ is no longer valid. This case is notoriously difficult for investigation. Until recently rather satisfactory results on the metal-insulator phase transition for the weakly disordered but strongly interacting electron system were obtained by the mapping of the problem onto the non-linear sigma-model that allows us the field-theoretic renormalization-group treatment. The related results for the scaling functions and critical exponents for different universality classes, as well as the description of those technically difficult methods can be found in [11]. We mention here only that these reuslts do not change the result that in 2D all states are localized no matter how weak the disorder is. From this point of view the recent discovery of the possible metal-insulator transition in 2D [12] looks quite surprising. The very recent phenomenological theory of this phenomenon based on the simple modification of the view of the β-function is given in [13]. This theory assumes the possible non-Fermi-liquid behaviour of the interacting two-dimensional electron gas that is of the questionable validity. However, the fact that people continue using the one-parameter scaling as the means for description of the conductivity related phenomena undoubtely gives additional credit to the theory discussed in this paper.

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