The Landau-Ginzburg/Calabi-Yau Phase Transition

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In this article we shall explain how the Abelian $\mathcal{N} = 2$ twisted supergauge theory in $1 + 1$ dimensions shows phase-transition in the very precise sense of the moduli space of the theory changing discontinuously as a function of the Fayet-Illiopoulous parameter (r) and the topological θ-term. This model allows for exact evaluation of renormalization effect on the critical point. It also gives a very analytically controlled scenario of seeing the two most important features of the liquid-vapour transition that the two phases have the "same" symmetries (here in a very precise sense) and there being the possibility of going around the critical point and thus transiting between the phases without encountering any singularity. The analogy is so compelling that the discoverer of this profound effect, Edward Witten, on page 29 of his revolutionary paper [1] comments, *"..like liquid and gas, Calabi-Yau and Landau-Ginzburg look like different phases.."*

A MOTIVATIONAL NUMEROLOGY

By a Landau-Ginzburg theory one would generically mean any theory with an unique classical ground state and that should be a degenerate critical point. There have been many reasons, stemming from surprising numerical coincidences, to believe that there should be a natural relation between (superconformal) Landau-Ginzburg theories and non-linear sigma models (with Calabi-Yau target manifolds) and thats what led to the initial path-breaking progress in this direction. (which eventually came to be known as "Gepner Models" in algebraic-geometry literature)

Here we list two very famous and deep results about these two aforementioned theories and notice one such coincidence,

- Consider the Landau-Ginzburg theory with a single chiral superfield X, with a superpotential $W(X)$ and a Kahler potential $K(X,\bar{X})$ with the action, $S = \int d^2 z d^4\theta K(X,\bar{X}) + \left(\int d^z d^2\theta W(X) + c.c\right)$

  It was shown in a series of seminal works by Vafa, Lerche and Warner that if the superpotential is of the form $W(X) = X^{P+2}$ for some $P \in \mathbb{Z}^+$ then at its IR fixed point the central charge of this theory is $\frac{3}{1+\frac{2}{P}}$. (...it is known that unitary $\mathcal{N} = 2$ superconformal field theories with central charge < 3 are minimal models and thats what facilitates these analyses..)

- It is known that a non-linear sigma model with a Calabi-Yau target manifold has vanishing $1-$loop beta-function and that it can be made to flow to a quantum CFT (at large Kahler moduli) and at that non-trivial IR fixed point it can be intuitvely argued (also known with some rigour) that its central charge is $= 3d$ where $d =$ the complex dimension of the Calabi-Yau manifold.

Now if one has say n CFTs with central charges $\{c_i\}_{i=1}^{i=n}$ then the central charge of the tensor product of these n theories is the sum $\sum_{i=1}^{i=n} c_i$ of the individual central charges. So if one takes the tensor product of n such Landau-Ginzburg theories then the central charge of the resulting theory will be $\sum_{i=1}^{i=n} \frac{3P_i}{P_i+2}$. One can infact choose all the Landau-Ginzburg theories to be identical and coming from a quintic potential (i.e $P_i = 3$ for all i) and consider 5 of them and then this sum becomes $= 9$. So if there has to be a conformally invariant non-linear sigma model which is "related" to this tensor product theory then there better exist a Calabi-Yau manifold at complex dimension 3.

Now it so happens that there does exist a very well-known Calabi-Yau manifold at 3 complex dimensions, called the "quintic 3-fold", and its the zero-set in $\mathbb{CP}^4$ of the degree 5 homogeneous polynomial, $z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5$. (..though it should be noted that Gepner went on to do a more detailed spectrum matching of the theories to accumulate evidence for such a correspondence to exist..)

A MOTIVATION FROM PATH-INTEGRAL

Consider the above Landau-Ginzburg theory (where the Calabi-Yau correspondence seemed possible) i.e with 5 superfields and all $P_i = 3$. Then the superpotential is $W = \sum_{i=1}^{5} X_i^5$. Then one can see sort of a heuristic reason why this Landau-Ginzburg theory will see the quintic-3-fold - at least in a region of the moduli space where the Kahler potential can be ignored. Then one is effectively looking at the following partition function,

$$Z = \int \prod_{i=1}^{5} \mathcal{D}X_i e^{i\int d^2z d^2\theta (\sum_{i=1}^{5} X_i^5 + c.c)}$$

On this path-integral one does the change of variables, $\xi_1 = X_1^5$ and $\xi_i = \frac{X_i}{X_1}$ (for $i = 2, .., 5$). Then the superpotential reads as, $W = \xi_1[1 + \xi_2^5 + \xi_3^5 + \xi_4^5 + \xi_5^5]$.

The important thing to note is that for this change of variables, the Jacobian is a constant $= 5$ and hence the partition function becomes,

$$Z \sim \int \prod_{i=1}^{5} \mathcal{D}\xi_i e^{i\int d^2z d^2\theta (\xi_1[1+\xi_2^5+\xi_3^5+\xi_4^5+\xi_5^5])}$$

Now the above is proportional to the delta function, $\delta(1 + \xi_2^5 + \xi_3^5 + \xi_4^5 + \xi_5^5)$

One can then easily see that if $(X_1, X_2, X_3, X_4, X_5)$ are coordinates on $\mathbb{C}^5$ then the moduli space to which the above delta function restricts the fields to is the same as the zero-set of the quintic $3-$fold in local coordinates $(\xi_2, \xi_3, \xi_4, \xi_5)$ in that open-set of $\mathbb{CP}^4$ where $X_1 \neq 0$. One observes that there is an invariance in the definition on rescalings of the form $X \mapsto Xe^{\frac{2\pi i}{5}}$ and hence the theory propagating on this Calabi-Yau manifold not exactly equivalent to the Landau-Ginzburg theory one started with but to an orbifold of it.

Some important points that need to be noted about the above argument are,

- It needs some detailed justification as to when the contribution of the Kahler potential can be ignored and that the above "semi-classical" argument gains more meaning and this isn't completely well-understood since it is related to taking the large gauge coupling limit but strong circumstantial evidence exits.

- The crucial thing that allowed for the localization of the moduli space of the theory is the fact that the Jacobian turned out to be a constant. In a more general scenario of arbitrary P_i this constancy of the Jacobian would turn out to be equivalent to the condition that the first Chern class of the zero-set of the superpotential is trivial and hence if and when the localization does happen its always to a Calabi-Yau non-linear sigma model.

Given these motivations a tantalizing possibility opens up of being actually able to show such a dramatic phenomenon of "phase-transition" of one CFT into another. Thus the search began for an interpolating theory and it lead to a much general construction by Edward Witten, some of which will be described in this article.

DEFINING THE THEORY

The theory we will be looking at is a twisted $\mathcal{N} = 2$ supersymmetric Abelian gauge field theory in $1 + 1$ dimensions. It suffices to look at super-renormalizable theories since they are simple representatives of their universality classes.

The schematic definition

The Lagrangian we will be looking at can be heuristically thought to consist of four parts as follows,

$$L = L_{\text{scalar chiral kinetic}} + L_{\text{superpotential}} + L_{\text{twisted gauge kinetic}} + L_{\text{topological D},\theta}$$

where we also choose an Abelian gauge group $U(1)^s$ and a shall be the index summing over 1 to s.

Writing the above in terms of $\mathcal{N} = 1$, $1 + 3$ dimensional superfields, Φ and V, (..we will eventually define the dimensional reduction to $1 + 1$ dimensions..)

- $L_{\text{scalar chiral kinetic}} = \int d^2y d^4\theta \sum_i \bar{\Phi}_i e^{2\sum_a Q_{i,a} V_a} \Phi_i$
- $L_{\text{superpotential}} = -\int d^2y d\theta^+ d\theta^- W(\Phi_i)|_{\bar{\theta}^+ = \bar{\theta}^- = 0} + h.c$
- $L_{\text{twisted gauge kinetic}} = -\sum_a \frac{1}{4e_a^2} \int d^2y d^4\theta \bar{\Sigma}_a \Sigma_a$
- $L_{\text{topological D},\theta} = -\sum_a r_a \int d^2y D_a + \frac{\theta}{2\pi} \int d^2y dv$

,

- Φ_i are a a bunch of scalar chiral superfields which in the $y-\theta$ coordinates ($y^\mu = x^\mu + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$) is given as,

$$\Phi(x,\theta) = \phi(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^\alpha\theta_\alpha F(y)$$

with ϕ, ψ and F being the scalar, fermionic and the auxiliary field components of the scalar chiral multiplet.

- $Q_{i,a}$ is the charge of the Φ_i under the $U(1)_a$ factor and e_a is the gauge coupling for each $U(1)_a$ factor.

- For each a (i.e each factor of $U(1)$ in the gauge group) the V_a is of the following form also in the above coordinates,

$$V = -\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}v_\mu + i\theta^\alpha\theta_\alpha\bar{\theta}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} - i\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\theta^\alpha\lambda_\alpha + \frac{1}{2}\theta^\alpha\theta_\alpha\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}D$$

where v_μ, λ and D are the gauge field, fermionic and the auxilliary components of the vector gauge multiplet. This also defines the D in the $L_{\text{topological D},\theta}$ term.

- $\Sigma = \frac{1}{2\sqrt{2}}\{\bar{\mathcal{D}}_+, \mathcal{D}_-\}$ is the twisted chiral superfield which is the field strength of the supergauge field in $1+1$ dimensions (..being in $1+1$ dimensions allows for this new possibility instead of the conventional Yang-Mill's curvature term..). Here $\mathcal{D}$ is the gauge superderivative and $+$, $-$ refer to the 1 and the 2 spatial direction in the $3+1$ dimensions from where one is dimensionally reducing to get the theory in $1+1$ dimensions.

The explicit form of the Lagrangian after dimensional reduction

We call the time direction in $1+3$ dimensions as the 0 and using the labels 1, 2 and 3 for the spatial directions, we define the dimensional reduction to be making the fields living in $1+3$ to be constant along the 1 and the 2 directions. Hence all derivatives with respect to x^1 and x^2 (equivalently y^1 and y^2) will be dropped. One often redefines a new y^0 and y^1 as $y^0 = x^0$ and $y^1 = x^3$. One defines new $\pm$ fermionic components as $(\psi^1, \psi^2) = (\psi^-, \psi^+)$, $(\psi_1, \psi_2) = (\psi_-, \psi_+)$.

After the aforementioned dimensional reduction and redefinition it follows that,

- $$L_{\text{scalar chiral kinetic}} = \sum_i \int d^2y(-D_\rho\bar{\phi}_i D^\rho \phi_i + i\bar{\psi}_{-i}(D_0 + D_1)\psi_{-,i} + i\bar{\psi}_{+i}(D_0 - D_1)\psi_{+i} + |F_i|^2$$
$$- 2\sum_a \bar{\sigma}_a\sigma_a Q_{ia}^2\bar{\phi}_i\phi_i - \sqrt{2}\sum_a Q_{ia}(\bar{\sigma}_a\bar{\psi}_{+i}\psi_{-i} + \sigma_a\bar{\psi}_{-i}\psi_{+i}) + \sum_a D_a Q_{ia}\bar{\phi}_i\phi_i$$
$$- i\sqrt{2}\sum_a Q_{ia}\bar{\phi}_i(\psi_{-i}\lambda_{+a} - \psi_{+i}\lambda_{-a}) - i\sqrt{2}\sum_a Q_{ia}\phi_i(\bar{\lambda}_{-a}\bar{\psi}_{+i} - \bar{\lambda}_{+a}\bar{\psi}_{-i}))$$

- $L_{\text{superpotential}} = -\int d^2y\left(F_i\frac{\partial W}{\partial\phi_i} + \frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_{-i}\psi_{+j}\right) + h.c$
- $L_{\text{twisted gauge kinetic}} = \sum_a \frac{1}{e_a^2}\int d^2y\left(\frac{1}{2}v_{01,a}^2 + \frac{1}{2}D_a^2 + i\bar{\lambda}_{+a}(\partial_0 - \partial_1)\lambda_{+a} + i\bar{\lambda}_{-a}(\partial_0 + \partial_1)\lambda_{-a} - |\partial_\rho\sigma_a|^2\right)$
- $L_{\text{topological D},\theta} = -\sum_a r_a\int d^2yD_a + \frac{\theta}{2\pi}\int d^2ydv = \frac{it}{2\sqrt{2}}\int d^2yd\theta^+d\bar{\theta}^-\Sigma|_{\theta^-=\bar{\theta}^+=0} - \frac{i\bar{t}}{2\sqrt{2}}\int d^2yd\theta^-d\bar{\theta}^+\bar{\Sigma}|_{\theta^+=\bar{\theta}^-=0}$ where $t = ir + \frac{\theta}{2\pi}$

The integrand of the $L_{D,\theta}$ term is called the "twisted superpotential", denoted as $\tilde{W}$, (compare to the W defined earlier), and its extremely crucial to note that $\tilde{W}(\Sigma) \sim t\Sigma$. *This linearity has deep ramifications and will be discussed in different contexts in the sections titled, "A first look at the "singularity" at $r = 0$", "The renormalization shift of the critical value" and "The phase diagram".*

ANALYZING THE POTENTIAL FOR THE BOSONIC FIELDS

For the sake of analyzing the vacuum expectation value its only necessary to look at the bosonic fields and by carefully looking at the above Lagrangian one realizes that (thankfully!) not all the content is important and that the bosonic fields are responding only to the following potential (from now on assuming the gauge group to be just $U(1)$ and hence there is no more sum over a, the label for the different $U(1)$ factors) ,

$$U = \frac{1}{2e^2}D^2 + \sum_i |F_i|^2 + 2\bar{\sigma}\sigma\sum_i Q_i^2|\phi|^2$$

(equations of motion set the auxiliary (non-dynamical) fields D and F_i to, $F_i = \frac{\partial W}{\partial\phi_i}$ and $D = -e^2(\sum_i Q_i|\phi_i|^2 - r)$

The condition for the R-charge anomaly to cancel forces the sum of the $U(1)$ charges of the fields to be 0. In compliance with that we choose to have $n+1$ fields in the theory and n of them (say $\{S_i\}_{i=1}^{i=n}$) will have charge 1 and one more field (say P) will have charge $-n$. Further $U(1)$ gauge invariance forces the superpotential, W, to be of the form, $W = P.G(S_1, S_2, ..., S_n)$ where G is a homogeneous polynomial of degree n. One further chooses G to be such that its "transverse" that is all $S_i = 0$ is the only solution of the n equations, $\frac{\partial G}{\partial S_i} = 0$. (..among other technicalities it ensures that the zero-set of G in $\mathbb{CP}^{n-1}$ is a smooth algebraic variety..) Let us use the notation of s_i and p for the scalar part of the superfields, S_i and P. Then under the above specific choice of the superpotential, the earlier mentioned potential for the bosonic fields evaluates to,

$$U = |G(s_i)|^2 + |p|^2 \sum_i |\frac{\partial G}{\partial s_i}|^2 + \frac{1}{2e^2} D^2 + 2|\sigma|^2 (\sum_i |s_i|^2 + n^2 |p|^2)$$

and the equations of motion sets the auxiliary D field to $D = -e^2(\sum_i |s_i|^2 - n|p|^2 - r)$

Now we can look for the two "phases" of the system at asymptotically large values of $|r|$ through an analysis of the minima of the above U and in either limit we will justify why the classical analysis is an exact quantum result.

Effective low energy physics at $r \gg 0$

Since $r > 0$, clearly D can't be equal to 0 unless, some of the s_i are non-zero. But by the transversality condition of G, if some of the s_i are not zero then the term $|p|^2 \sum_i |\frac{\partial G}{\partial s_i}|^2$ term can be zero only if $p = 0$. So the vanishing of D sets a constraint on the scalar components as $\sum_i |s_i|^2 = r$.

Now since some of the s_i are non-zero and $p = 0$, the only way the last term of U can be 0 is when $\sigma = 0$. So the remaining constraint to get $U = 0$ is to set $G(s_i) = 0$.

So classically the moduli space of fields on which the potential minimizes for $r > 0$ is when the following four conditions are satisfied i.e, $p = \sigma = 0$ and $\sum_i |s_i|^2 = r$ and $G(s_i) = 0$. The two conditions are equivalent to that of having a non-linear sigma model but more is true. Since G is a homegenous polynomial of degree n in n variables, its zero-set is a Calabi-Yau manifold and hence the theory is that CFT mentioned at the beginning of this article, a non-linear sigma model with a Calabi-Yau target manifold. Thus the Calabi-Yau condition can be seen to emerge as a consequence of making the R-symmetry non-anomalous

Now note that the Kahler class of the $G = 0$ hypersurface is r and from the theory of non-linear sigma model one knows that in the $r \gg 0$ limit the theory is weakly coupled and hence in that limit the above classical analysis will become an exact quantum result and one then has a quantum CFT.

Effective low energy physics at $r \ll 0$

For $r < 0$, the vanishing of D clearly requires having $p \neq 0$. Since $p \neq 0$, and G is assumed to be transverse it follows that the vanishing of $|p|^2 \sum_i |\frac{\partial G}{\partial s_i}|^2$ requires all the $s_i = 0$. So the vanishing of D necessarily requires, $|p| = \sqrt{-rn}$. But by a gauge transformation one can get the argument of p to vanish.

Now if one goes to the $r \ll 0$ limit then the p fields are becoming infinitely massive and hence can be thought to be integrated out leaving behind a theory of massless fields s_i. And fluctuations of these massless fields is governed by a polynomial effective potential thought of as, $\sqrt{-r}\tilde{W}(s_i)$ and that is degenerate at its unique critical point (origin) and hence its a Landau-Ginzburg theory (an orbifolded one since inspite of giving a vev to the s_is, the homogeneity of $\tilde{W}$ preserves a remnant $\mathbb{Z}_n$ symmetry under the transformation, $s_i \mapsto \xi s_i$ where ξ is a n^{th} root of unity).

As stated in the opening section, there exists compelling evidences that in the IR the above Landau-Ginzburg theory renormalization flows to a quantum CFT and using the notation defined there the central charge count matches for the two candidate phases as, $\sum_{i=1}^{n} \left(\frac{3}{1+\frac{2}{P_i = n-2}} \right) = 3(d = n-2)$ (P_is are for a "diagonalized" basis for G and $P = 0$ in the Calabi-Yau phase). This degree of freedom matching is a way of making it precise as to why like liquid and vapour here too the two phases have the "same" symmetry, though there exists subtle geometric differences and the phases/moduli spaces are only not exactly isomorphic but are birationally equivalent as algebraic varieties.

A first look at the "singularity" at $r = 0$

From the above analysis it seems that there might be a "singularity" at $r = 0$ in going from the Landau-Ginzburg to Calabi-Yau models. This is unlike the normal notion of a singularity associated with a phase-transition since the familiar situations are invariably related to the infinite size limit of the substance or here of the world-sheet. But there still seems to emerge a singularity even if one is working on compact spaces. It is largely attributable to a failure of the effective compactness of the target space.

The problem begins when $s_i = p = 0$ since then U becomes constant and equal to $\frac{e^2r^2}{2}$ and hence one can go to arbitrarily large values of σ at no energy cost! One notes that this phenomenon of having a "flat direction" is a consequence of the fact that the classical twisted superpotential is linear in the twisted supergaugefield. Classically this flat direction is the reason for a singularity/criticality to exist in the theory since at $s_i = p = r = 0$, the ground state is singular/non-normalizable as the potential isn't diverging at infinity in field space. This is akin to the situation of the ground state becoming undefined in the $k = 0$ limit of the simple harmonic oscilator with $\frac{1}{2}kx^2$ potential.

But we shall now see that this classical critical point of $r = 0$ undergoes quantum modifications of two different kinds.

QUANTUM ENERGY OF THE TOPOLOGICAL TERM HELPS GO AROUND THE CRITICAL POINT

Quantum theoretically one knows that the θ-dependance of an $U(1)$ gauge theory in $1+1$ dimensions is given by, $\frac{e^2}{2}min_{n\in\mathbb{Z}}(n - \frac{\theta}{2\pi})^2$. Hence in the classically critical regime of $\sigma \to \infty$ the potential goes as $min_{n\in\mathbb{Z}}\frac{e^2}{2}|\tilde{t}|^2$ where $\tilde{t} = t + n$ with $n \in \mathbb{Z}$.

Hence this quantum effect makes the potential non-zero even for $r = 0$ as long as $\theta \neq 0$ and hence allows for an energy range such that the spectrum is discrete and the ground-state is normalizable where classically it wouldn't have been. Its effect on the phase-diagram will be stated in the section titled, "The Phase Diagram".

EXACT RENORMALIZATION SHIFT OF THE CRITICAL POINT

The singularity at $r = 0$ came from the region in the field space where $s_i = p = 0$ and $|\sigma|$ is large. One notes that its the linearity of the twisted chiral superpotential that ensures that Σ doesn't contribute to the potential energy of the system, except through providing mass to the chiral superfields in proportion to $|\sigma|^2$, as can be seen in the bosonic potential. We need to analyze the theory in its classical critical regime of all other fields vanishing but large σ.

Keeping in view the larger questions where this current phase structure analysis naturally comes up, let us slightly generalize this discussion and use the notation of $\{B_i\}_{i=1}^{n+1}$ to denote the $n+1$ fields, $\{P, S_1, S_2, .., S_n\}$ and let them have arbitrary $U(1)$ charges $\{q_i\}$ (..instead of the earlier assignment of $\{-n, 1, 1, ..\text{n times}.., 1\}$ consistent with R-charge anomaly cancellation..) The deviation from 0 of the expectation value of the $\bar{B}_i B_i$ comes from the $1-$loop correction to that. This is in turn coming from the quantum $2-$point correlation function, $\langle \bar{B}_i B_i \rangle$ where the classical mass of the B_i fields comes from the term $|\sigma|^2 \sum q_i^2 |B_i|^2$(...which upto rescalings of the definition of charge is the erstwhile term, $2|\sigma|^2(\sum_i |s_i|^2 + n^2|p|^2)$...),

$$\sum q_i \int^{\Lambda} \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 + q_i^2|\sigma|^2} = -\frac{1}{2\pi} \sum q_i ln|q|_i - \frac{1}{2\pi} (\sum q_i) ln \frac{|\sigma|}{\mu}$$

In the language of renormalization this is the $1-$loop redefinition of the twisted superpotential (classically which was $\tilde{W}(\Sigma) \sim t\Sigma$) to,

$$\tilde{W}_{effective}(\Sigma) = \left(t - \frac{i}{2\pi} \sum q_i ln|q_i| \right) \Sigma + \frac{i \sum q_i}{2\pi} \left(\Sigma - \Sigma ln \left(\frac{\Sigma}{\mu} \right) \right)$$

The above is interpreted to see that the singularity or the critical value of the system is neither at the classical value of $r = 0$ nor at the quantum mechanical value of $t = 0$ but it is actually at the the point,

$$t_{critical} = ir_{critical} + \frac{\theta_{critical}}{2\pi} = \frac{i}{2\pi} \sum_i q_i ln|q_i|$$

This is effectively a renormalization shift (to say $r = r_0 \neq 0, \theta = 0$) of the value where the criticality happens. One further notes that this is an *exact* result and that there are no higher order corrections can be argued from either the holomorphy principles of Seiberg or by doing a power counting of the higher loop diagrams to show that they are vanishing in the large $|\sigma|$ limit,

THE PHASE DIAGRAM

As in the original case of our interest, if $\sum q_i = 0$ (the same condition as for R-charge anomaly cancellation!) one sees that there are no logarithmic renormalization corrections to the twisted superpotential and hence the $\tilde{W}_{effective}$ remains linear inΣ (as it was classically). Hence inspite of renormalization flow it remains possible for σ to go arbitrarily large without any cost to energy and thus giving rise to non-normalizable ground-states and that is the singular behaviour at the point of phase-transition.

But if $\sum q_i \neq 0$ then there is a $|log\sigma|^2$ divergence in the energy and that prevents any non-normalizable mode to ever develop for any value of $r \in \mathbb{R}$ and $\theta \in [0, 2\pi]$ and hence there are no singularities in the vacuum behaviour on the t-cylinder (..the "phase-space"..). So that allows for a smooth quantum interpolation between non-homeomorphic classical geometries! (but at the cost of having an anomalous R-charge)

Thus in our original Landau-Ginzburg$(r \ll 0)$/Calabi-Yau $(r \gg 0)$ phase-transition case the condition $t_{critical} = 0$ is tantamount to renormalization shifting the critical point to $r = r_0 = \frac{n}{2\pi} ln(n) \neq 0, \theta = 0$ (as defined earlier). Hence this topological parameter, θ allows one to "go around" (like in liquid-vapour transition!) even the renormalization flow corrected critical point by choosing a path in the phase space which keeps to non-zero values of θ around $r = r_0$ and hence avoids the singularity. Thus by choosing such a path, one is most probably not just interpolating between the two phases (evidenced to be CFTs) at the asymptotic values of r but is doing so via a single parameter family of CFTs. Being able to exactly prove this belief (which for now is supported by renormalization theory) can have deep ramifications in physics and mathematics.

PROPOSALS FOR FUTURE RESEARCH DIRECTIONS

To the best of my knowledge the following three directions from here are still unexplored. *Firstly*, this construction apart from its pedagogic value as an exact model of phase-transitions, when generalized for arbitrary homogeneous polynomial G of degree k, is infact quite an analytically controllable demonstration of the physically very intutitive Zamolodchikov's $c-$theorem, $c_{IR} \leq c_{UV}$. In recent times there have been attempts to prove the analogous a-theorem in four dimensions (like work by Zohar Komargodski and Adam Schwimmer, arXiv:1107.3987). It might be a fruitful venture to see if an analogous phase-transition model works there too. *Secondly*, its still not fully convincingly proven that the IR flows of this theory at asymoptotic values of r is indeed to CFTs. Such efforts might help produce more exact results/evidence/proofs of mirror symmetry. *Lastly*, it might be worthwhile to try to generalize the above construction to other supergauge theories.

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