Phase Transitions in Strongly-Interacting Matter and Cold Atomic Systems

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Abstract

While of vastly different energy scales, cold atomic gases and dense nuclear matter possess a surprising number of similarities in their qualitative behavior and phase structure. The novel properties of these systems arise from the often strong interactions of many degrees of freedom, which leads to a very rich, and complex, phase structure. Recent experimental advances have given researchers an unprecedented level of control over atomic systems and allowed for a detailed study of the phase transitions of condensed matter systems, while bigger and better particle accelerators continue to push the frontier of our knowledge of quark and hadron physics. The results of these studies demonstrate some striking similarities including a transition from "fundamental" (quarks and BCS atomic pairs) to composite (hadrons and BEC atomic pairs) degrees of freedom and nearly inviscid flow dynamics. In this paper we study these similarities and discuss ways in which the two fields may inform one another.

1 Introduction

The past twenty years has seen a remarkable advance in experimental techniques in condensed matter physics, and particularly in our ability to the prepare ultracold systems. The ability to routinely achieve temperatures in the nanokelvin range has greatly increased our capacity to study both fermionic and bosonic systems in their non-classical limits. Of course, degenerate Fermi gasses and Bose-Einstein condensates possess vastly different properties, which follow directly from their respective statistics. However, in the last ten years it has become possible to connect these two systems and convert directly between them by means of a Feshbach resonance [1, 2]. These advances have allowed researchers to probe the very rich phase diagram of cold atomic systems with a precision of which theoreticians could previously only dream.

Interestingly, the weak-to-strong coupling transition in cold atomic fermi gases which transforms BCS atomic pairs into BEC pairs, is qualitatively very similar to a transition that occurs in nuclear matter. In the last decade, while atomic physics researchers were busy achieving nanokelvin temperatures, a new generation of particle accelerators were creating a new, high temperature, high density, state of matter known as the quark-gluon plasma [3]. This state is characterized by deconfined quarks and gluons which move freely, and is thought to represent the state of the universe very shortly after the big bang, and may also exist in the cores of neutron stars [4, 5]. This exotic state of matter is a far cry from the confinement observed under standard conditions, and therefore a phase transition connecting the two states must exist at some critical temperature and density [6]. The confined phase, in which protons and neutrons consist of three bound quarks, is analagous in some sense to the tightly bound BEC atomic pairs that form in the unitarity limit of a Feshbach resonance.

In addition to the strong-to-weak coupling transition, cold atomic systems and quark-gluon plasmas also exhibit nearly inviscid hydrodynamic flows [7, 8]. This property gives experimentalists the opportunity to test the proposed universal lower bound on the viscosity-to-entropy density [9]

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \tag{1}$$

This ratio, which is $\sim 10^{-12}\mathrm{K} \cdot \mathrm{s}$, is drastically lower than the values typically observed in nature, but the nearly ideal hydrodynamics realized in both cold atomic systems and quark-gluon plasmas provide a rare opportunity to approach, or perhaps violate, the bound. There are also interesting analogies between recent work in the BEC pairing of unequal mixtures of fermions and the pairing of quarks of unequal masses (e.g. up/down with strange), the latter of which has posed some theoretical obstacles, though we will not discuss this particular connection in this paper. Thus, despite the fact that these systems are separated in energy by an astounding 21 orders of magnitude ($T_c \sim 10^{-9}$K and $T_h \sim 10^{12}$K), they exhibit some intriguing qualitative similarities, which will be the focus of this paper.

This paper is organized as follows. In Sec. 2 we outline the properties of cold atomic systems and the Feshbach resonance which enables one to pass continuously from the weak-to-strong coupling limit. In Sec. 3 we turn to nuclear matter and give an overview of its properties and phase structure and in particular, the properties of the quark-gluon plasma. In section 4 we discuss the qualitative similarities of these two systems including the weak-to-strong coupling transition, nearly ideal hydrodynamics, and BEC pairing. Finally, in section 5 we conclude by summarizing our observations and commenting on ways in which the fields may inform one another in future research.

2 Properties of Cold Atomic Gases

The use of cold atomic gases to simulate condensed matter systems has virtually exploded since their realization in the late 1990s [2, 10, 11]. Degenerate fermi gases and Bose-Einstein condensates, systems which had been studied for decades by theoreticians, were finally experimentally accessible, and so many theories which had henceforth gone untested, were now testable. In particular, the prospect of using the so-called Feshbach resonance to tune the effective two-body interaction, which was predicted some forty years previously (1958) by Herman Feshbach, in the context of nuclear physics, was finally realized in 1998 by Inouye, *et. al* at MIT [1, 2]. This method involves the manipulation of different hyperfine states by means of an external magnetic field so that, at resonance, the energy of the molecular state is equal to that of free atoms. The result is an s-wave scattering length which diverges, spanning an infinite range of values within a finite range of applied magnetic field, as shown in figure 1.

The s-wave scattering length is a parameter often referred to in scattering theory, and is particularly useful in the present context. Essentially, it is a scaled measure of the interaction strength, which we assume to be of the form

$$U(\mathbf{r}) = U_0 \delta(\mathbf{r}) \tag{2}$$

The s-wave scattering length is then defined as [12]

$$a = \frac{m}{4\pi\hbar^2} U_0 \tag{3}$$

The utility of the Feshbach resonance is its ability to tune the interaction strength arbitrarily. That is, for any desired interaction parameter U_0, there exists a magnetic field which produces it. Moreover, as shown in figure 1, the resonance is so sharp that, for ^{23}Na atoms, virtually the entire range of interaction parameters is obtainable with a variation of only 15 G in the external field (900-915 G). In particular, we can easily transition from a weak-coupling regime, away from resonance, to a strong-coupling regime, near resonance, drastically changing the physical properties of the system. We will now analyze this transition by looking at these two regimes.

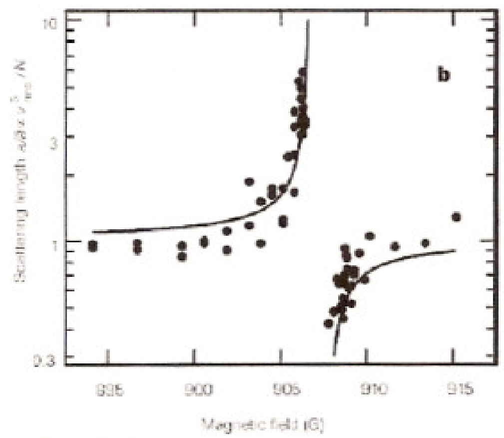

Figure 1: S. Inouye's observation of the normalized scattering length of ^{23}Na atoms as a function of applied magnetic field. [2]

2.1 Weak-Coupling (BCS) Regime

Let us consider the case of a weak two-body attraction, where $a < 0$. This situation is analagous to the BCS model of superconductivity in which electrons are the constituent particles (in place of atoms), and the interaction is phonon-mediated rather than due to the hyperfine splitting [13]. It is now well-known, as shown by Leon Cooper, that the Fermi sea is unstable to even an arbitrarily weak two-body attraction, and that fermions (^{23}Na atoms, for example) near the Fermi surface will form weakly bound Cooper pairs with energy [14, 15]

$$E_{pair} \approx 2\epsilon_F - 2\hbar\omega_c e^{2/\rho(\epsilon_F)U_0} \tag{4}$$

where ω_c is a characteristic cut-off frequency of the system and $\rho(\epsilon_F)$ is the density of states at the Fermi surface. Note that since $U_0 < 0$ the pair formation gives an exponentially supressed energy benefit, but one that is nonetheless non-zero. Thus, so-called BCS pairs of cold atoms will form, but due to the weak coupling, these pairs will be spatially extended over several inter-atomic spacings:

$$\xi_0 \sim \frac{\hbar v_F}{k_B T} \tag{5}$$

where v_F if the Fermi velocity of the atoms. For a typical experiment, such as that performed by Greiner, *et al.*, involving ^{40}K atoms we might have $T \sim 100$nK and atomic cloud densities of $n \sim 10^{21}/m^3$ so that $\xi_0 \sim 1\mu$m while the inter-atomic spacing is $n^{-1/3} \sim 0.1\mu$m [16, 17]. We should note that this formation of Cooper pairs is strictly a many-body effect, as the two-body problem does not have a bound state solution for an arbitrarily small interaction potential. On the other hand, for a sufficiently strong attraction the two-body problem does posess a bound state solution in which the particles can be treated as a single (bosonic, or BEC) molecule. It is now this regime to which we turn.

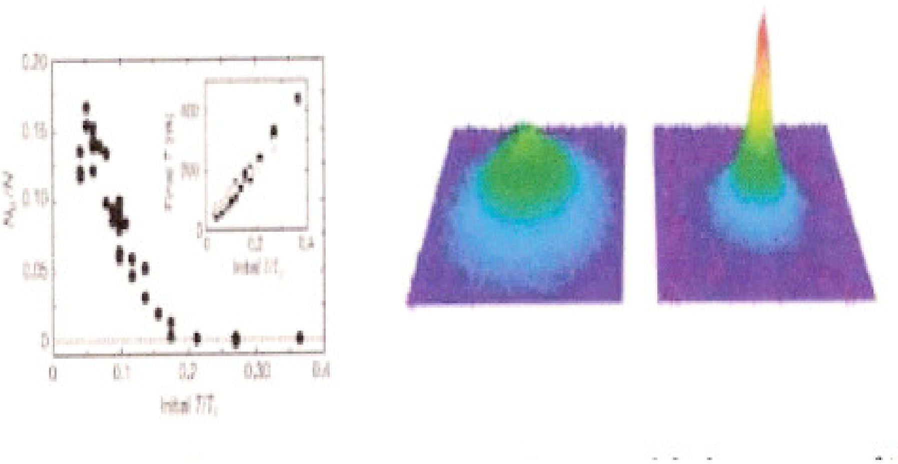

Figure 2: At left, Greiner's dependence of the condensate fraction and final temperature of the atom-molecule mixture on the initial temperature (T/T_F) of the Fermi gas ^{40}K. At right, a surface plot of the optical density for the (bosonic) molecule created by sweeping through the Feshbach resonance with an initial temperature of $0.19T_F$ ($0.06T_F$) for the left (right) image [16].

2.2 Strong-Coupling (BEC) Regime

By sweeping the external magnetic field near the Feshbach resonance, we can change the interaction strength from weak to strong and greatly alter the properties of the atomic system. This phenomenon was first experimentally realized by a number of groups in 2003, and quickly revolutionized the field [16, 18, 19]. As shown in the prior section, and as was known since the mid 1950s, a fermionic system will form weakly-bound BCS pairs in the presence of an arbitrarly small attractive interaction. However, for a strong attractive interaction the fermions instead form tightly-bound composite "molecules" which may be themselves treated as bosonic particles. The formation of such BEC pairs from the weakly-bound BCS pairs of a "normal" Fermi gas has been observed in both ^{6}Li and ^{40}K [16, 18, 19]. The formation of these pairs is demonstrated unequivocally by measuring the spatial variation in the gas density on the BEC side of the resonance. As shown in figure 2, the profile is decidedly non-thermal and is characteristic of a Bose-Einstein condensate. But it is certainly not the fermionic atoms themselves which have condensed, and therefore they must have formed bosonic molecules which make up the observed condensate.

Thus, as the interaction strength is varied from weak to strong, the fermionic atoms pass from a weakly bound state (or, indeed, a free state if we set $a = 0$) to a strongly-coupled molecular state in which the "fundamental" degrees of freedom of the system are no longer individual atoms, but composite bosons. This state of affairs is reminiscent of the formation of composite hadrons such as protons and neutrons, from their fermionic (quark) constituents, which occurred shortly after the big bang, and which occurs today in the aftermath of the high energy collisions in particle accelerators. Indeed, the similarities between these two vastly different states of matter are intriguing, and in order to study them further we now turn to a brief overview of nuclear matter and its properties.

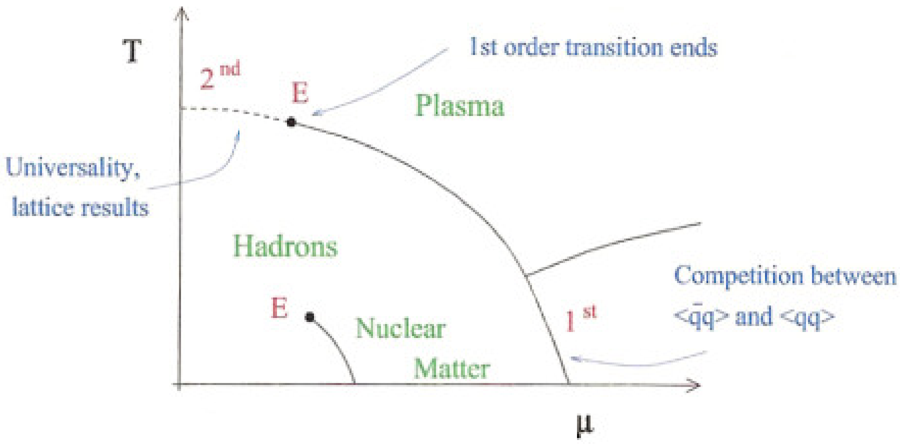

Figure 3: This figure shows the phase diagram of strongly-interacting matter obtained from a mean field treatment of chiral symmetry breaking and color superconductivity in QCD with two flavors [20].

3 Nuclear Matter Phase Structure

The phase structure of nuclear matter is decidedly more difficult to probe than that of cold atomic systems because it is impossible to investigate its behavior at the high temperatures and densities of interest (i.e. those at which phase transitions take place) in a controlled laboratory environment. Of course, the same was true of cold atomic systems only 20 years ago, so perhaps we may hope for techological advancements which similarly render the regime of nuclear transitions more accessible. And indeed, in the last 20 years we have made great progress in this area, designing and building particle accelerators which are capable of producing the energies and beam densities necessary to probe the fundamental structure of nuclear matter. Nonetheless, detailed emperical knowledge of the high temperature and density properties of nuclear matter (i.e. the "quark-gluon" plasma) is still lacking and we hope that a new generation of accelerators will shed greater light on this fascinating problem. In fact, there is also hope given by the advances made in cold atom research, as the Feshbach resonance enables researchers to create strongly-correlated systems which simulate the behavior of nuclear matter.

Fortunately, quantum chromodynamics (QCD), the theory of the strong nuclear force, is well-known, if not easily solved, and an analysis of the theory may give some insight into the behavior of nuclear matter under extreme conditions. As with any matter, upon increasing the density and/or temperature of nuclear matter sufficiently, the "fundamental" components, that is, the quarks and gluons, will be liberated and form a plasma, in precise analogy to the formation of an electronic plasma in the sun, for example. The nature of this transition will depend on the details of the system, and a very good overview of the factors involved is presented by Schäfer [20].

As the diagram shows, at intermediate chemical potentials (densities), there is a first order phase transition between the quark-gluon plasma and hadronic bound states (i.e. protons and neutrons) as the temperature is lowered. Interestingly, universality and lattice

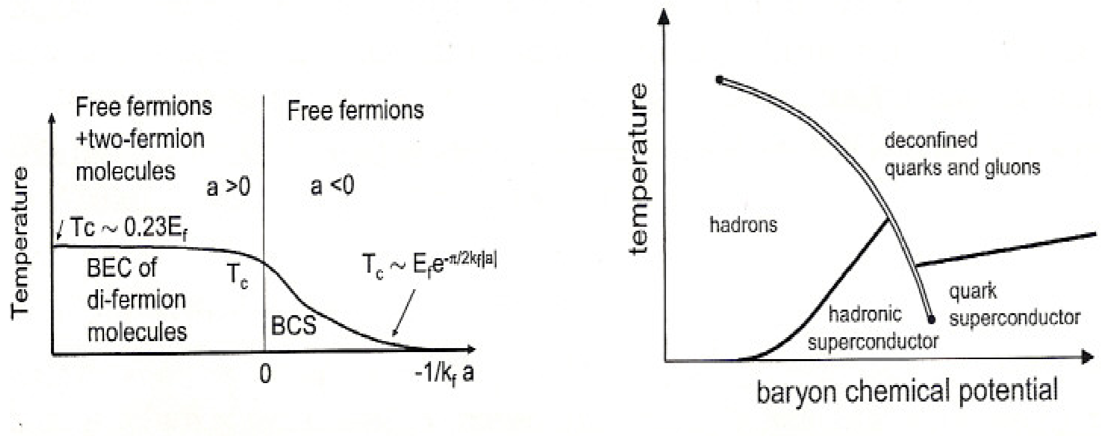

Figure 4: This figure shows approximate phase diagrams of strongly-interacting and cold atomic matter. The parameter space is temperature and inverse scattering length (normalized by the Fermi wavenumber) for cold atomic systems and temperature and baryon chemical potential for strongly-interacting systems [21].

QCD calculations indicate that at low enough densities this transition turns into a second order transition, although we will not pursue this fact further here. The important fact, for our consideration, is the fact that below a certain critical density there is a phase transition between the weakly coupled quark gluon plasma and strongly coupled hadrons.

4 Cold Atoms and Nuclear Matter

As we have seen above, both cold atomic systems and nuclear matter undergo strong-to-weak coupling transitions in which spatially separated BCS pairs are converted into BEC molecules. If the hadronic particles (protons and neutrons) were composed of only two quarks then the analogy with cold atoms would be exact. This difference suggests that we may not expect the similarities between these systems to be particularly profound, such as relating to a shared similarity group [21]. However, we need not allow this fact to discourage us, as there may nonetheless exist fruitful comparisons between the two systems. We now discuss the analogies that may be made between these systems by analyzing the various states of each.

4.1 Weak-Coupling Limit

The best known example of a many-effect in the weak-coupling limit is BCS pairing in conventional superconductors. As Bardeen, Cooper, and Schriefer showed, the formation of BCS pairs of electrons give rise to superconductivity [13]. Similarly, as Leggett and others showed, the formation of BCS pairs of cold atoms (e.g. ^{3}He) give rise to superfluidity [22]. Extending these observations to nuclear matter, we expect that BCS pairing of quarks, which will occur in the weak-coupling regime, to give rise to a phenomena that we might call *color superconductivity* [23, 24]. In this state the BCS quark pairs condense into the system ground state and the system is described by a macroscopic wave function. Indeed, as Braun-Munzinger and Wambach show, a number of different color superconducting states are expected, some with "normal" state quark matter, and others in which the chiral symmetry of the system is spontaneously broken [23]. In fact, some of these color superconducting states are thought to exist not only transiently in high-energy collisions, but also in the cores of neutron stars [5].

4.2 Strong-Coupling Limit

In the strong coupling limit, as described above, both cold atoms and quarks form tightly bound states. An important difference exists, however, between the systems in this limit. The atomic BEC pairs are bosonic and can therefore condense into a superfluid phase, as occurs in liquid ^{3}He. However, due to the fact that quarks bind into 3-quark composites (i.e. protons and neutrons), the "molecules" are themselves fermionic and therefore cannot directly form Bose-Einstein condensates. This distinction is one reason that multiple color superconducting states exist, as noted above, and from the nuclear matter's phase diagram we note that at sufficiently low temperatures and high chemical potentials it is in fact mesons, not nucleons, which tend to form [23].

4.3 Nearly Ideal Hydrodynamics

Another intriguing similarity between cold atomic systems and the quark-gluon plasma is their shared property of nearly inviscid flow near unitarity ($a = \infty$). That is, in this limit, both systems are characterized by abnormally small viscosities. The viscosity of cold atomic systems has been studied experimentally by a number of groups, and the results demonstrate, as shown in the accompanying figure, a viscosity to entropy density that decreases with decreasing temperature, apparently approaching a constant value as $T \to 0$ [25, 26]. We should note that some theoretical calculations suggest that the ratio, rather than approaching a constant value, actually trends upward again at very low temperatures, with a minimum existing at a temperature $\sim T_c$. In either case, this result is important in part because it gives credence to a proposed universal bound on this ratio, which was proposed recently by Kovtun [9].

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \tag{6}$$

Experimental results give a ratio roughly ten times the theoretical minimum, and a value significantly lower than nearly any other experimentally observed system. As Baym point out, classically, since the entropy density is proportional to the density of particles ($s \sim n$) and the viscosity is roughly $\eta \sim nmv^2\tau$, where τ is a characteristic relaxation time, we expect that (inserting a factor of k_B for dimensional consistency) [21]:

$$\frac{\eta}{s} \sim \frac{mv^2\tau}{k_B} \sim \frac{(mv)(v\tau)}{k_B} \sim \frac{pl}{k_B} \tag{7}$$

where p is the average momentum and l the mean free path. Setting this equal to the proposed bound we find

$$pl \sim \hbar \tag{8}$$

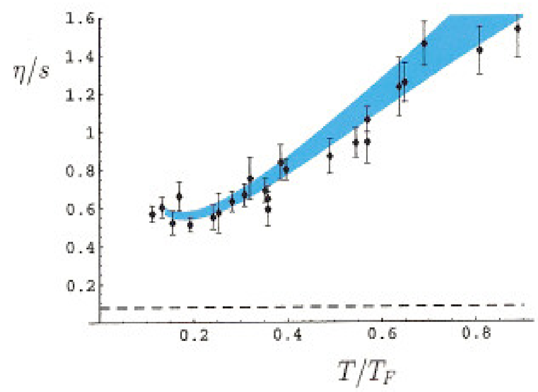

Figure 5: Plot of the viscosity to entropy density ratio of a cold atomic gas in the unitarity limit (in units where $\hbar = k_B = 1$). The dashed line is Kovtun's conjectured lower bound $\eta/s = 1/4\pi$, while the shaded region is a systematic error estimate based on the contribution to $\dot{E}$ due to atoms outside a surface of unit optical depth [28].

which is simply a statement of the Heisenberg uncertainty principal. Indeed, physically the proposed bound amounts to the condition that the mean free path of a particle is at least as big as the inter-particle spacing [21].

Similarly, the proposed quark-gluon plasma which has been produced at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory has flow properties which imply a very small viscosity to entropy density ratio ($\eta/s \ll 1$) near the critical temperature (i.e. near unitarity) [8]. Unfortunately, as is characteristic of strongly-coupled systems, currently there is no consistent theoretical method to reliably compute this ratio near unitarity, which explains why Kovtun's bound has not been proven rigerously. However, weak-coupling calculations clearly demonstrate a ratio which decreases with increasing coupling, in accordance with both Kovtun's conjecture and the experimental results outlined above [27].

5 Conclusions

We have seen that despite the fact that strongly-interacting nuclear matter and cold atomic systems are among the hottest and coldest systems in the universe, separated by roughly 21 orders of magnitude, they share a number of common properties due to their many-body nature. The presence of a weak-to-strong coupling transition and nearly ideal hydrodynamics near unitarity would be intriguing, even if the similarities went no further. However, the recent advances in condensed matter systems, in particular the ability to use cold atomic gases to simulate strongly-interacting matter has opened up areas of research which were previously unaccessible. These new studies, which are only now in their infancy, combined with the data from our newest, most energetic, particle accelerators promise a plethora of exciting results in the near future which will both test existing theories and provide inspiration for new ones.

Perhaps the most interesting feature that these two systems share is that of the strongly-correlated, many-body behavior, which continues to frustrate theorists. Alas, the fact that two systems share in an insoluble problem does not make it more easily solved. However, the unique control which experimentalists can now exact over cold atomic systems will allow us to discern more fully than ever before the richness of strongly-correlated systems. We hope that this new information will infuse the fields with new ideas which will both contribute to our understanding of these complex systems and the methods by which we may more accurately describe them.

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