Phase Transition in Collective Motion

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May 4, 2008

Abstract

There has been a high interest in studying the collective behavior of organisms in recent years. When the density of living systems is increased, a phase transition from a disordered state into an ordered state occurs: i.e. units, which move in random directions below the transition, move together in the approximately same manner or direction. Several models and experiments are reviewed in this essay.

1. Introduction

A high interest in studying the collective behavior of organisms has been raised in recent years. When the density of living systems is increased, a phase transition from a disordered state into an ordered state occurs – units, which move in random directions below the transition, move together in the approximately same manner or direction (Fig. 1).

Fig.1. Fish schools [Photo: Norbert Wu, 1999]

This collective motion emerges at all sizes, from cells to whales. Several assumptions based on evolutionary functions are proposed to explain why organisms tend to behave similarly, such as increasing survivorship, mating, food finding, etc. [Parrish *et al*., 1999]. Besides evolutionary assumptions, several models based on self-propelled particles have been developed. Also, a few experiments were successfully carried out under laboratory conditions recently. Models and experiments are reviewed in the following sections.

2. Models and simulations

The self-propelled particle models proposed in about last 10 years mainly fall into two categories: one is assuming the particles have simple short-range interaction with some random noise, and the other one employs more complicated interactions.

2.1 *Simple short-range interaction with noise added*

Vicsek *et al*. introduced their model in 1995 with a simple rule that particles were propelled with a constant absolute velocity and the average direction of motion in a particle's neighborhood was assumed at each time step with some random noise added [vicsek *et al*.1995]. In simulations, Vicsek *et al*. assumed that N particles were randomly distributed in a square shaped cell at initial time ($t = 0$), and they had the same absolute velocity v and random distribution of velocity directions θ. The position of the ith particle is

$$\vec{x}_i(t+\Delta t) = \vec{x}_i(t) + \vec{v}_i(t)\Delta t, \tag{1}$$

where the simultaneous velocity $\vec{v}_i(t)$ was determined at each step and Δt was time step. $\vec{v}_i(t+\Delta t)$ here has an absolute value v, and a direction given by $\theta(t+\Delta t)$ which is determined by

$$\theta(t+\Delta t) = \langle \theta(t) \rangle_r + \Delta\theta, \tag{2}$$

where $\langle\theta(t)\rangle_r$ was the average direction of particles within a circle of radius r (interaction range) surrounding a particular particle, and $\Delta\theta$, presenting noise, was a random number chosen in the interval $[-\eta/2, \eta/2]$. Figure 2 (a - d) shows the velocity field with various noise parameters η and density $\rho = N / L$. Fig. 2(d) with high density and low noise demonstrates the most interesting result – most particles have ordered motion in approximately the same direction. The absolute value of the average normalized velocity was also determined and taken as an order parameter

$$v_a = \frac{1}{Nd} \left| \sum_{i=1}^{N} \vec{v}_i \right|. \tag{3}$$

When the velocity of particles was randomly distributed initially $v_a = 0$, and when the motion of all the particles became ordered $v_a = 1$. Figure 3 (a) demonstrates v_a as a function of η at fixed density ρ with different sample size N, and figure 3(b) shows behavior of v_a as density ρ changes at fixed noise η.

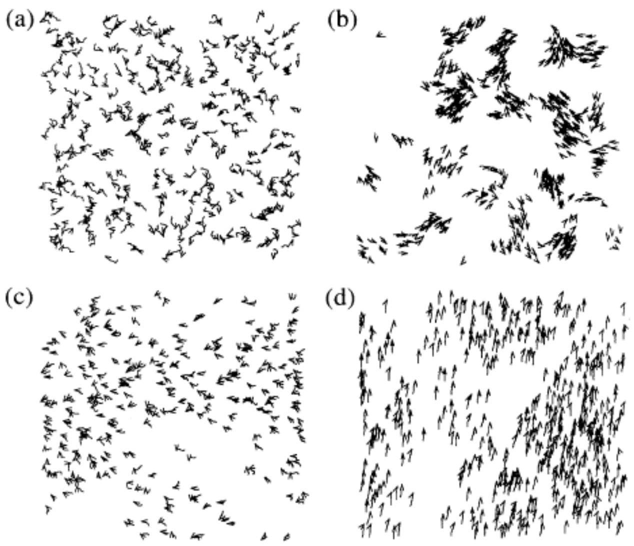

Fig. 2 Velocity field with different noise parameters η and density ρ. N = 300 in each case. (a) t = 0, L = 7, η =2.0. (b) L = 25, η = 0.1. (c) L = 7, η = 2.0. (d) L = 5, η = 0.1

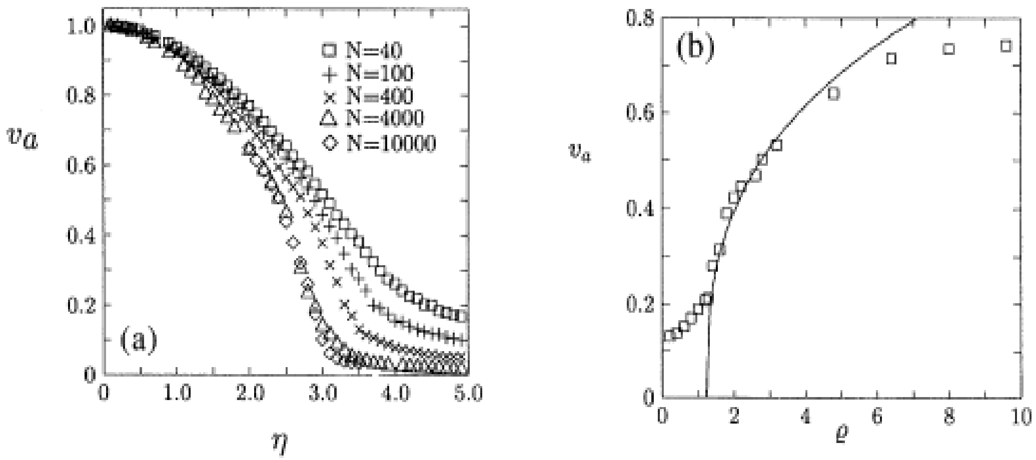

Fig. 3 (a) the average normalized velocity as a function of noise. (b) the average normalized velocity as a function of density

Since the behavior of v_a is similar to that of the order parameter in equilibrium systems, Vicsek *et al*. assumed that this kinetic phase transition was analogous to the phase transition in equilibrium systems,

$$v_a \sim [\eta_c(\rho)-\eta]^{\beta}, \text{ and } v_a \sim [\rho-\rho_c(\eta)]^{\delta} \tag{4}$$

where $\eta_c(\rho)$ and $\rho_c(\eta)$ are critical points. $\beta = 0.45\pm0.07$ and $\delta = 0.35\pm0.06$ were obtained by linearly fitting data in plot of dependence of ln v_a on $\ln([\eta_c(L)-\eta]/\eta_c(L))$ and $\ln([\rho-\rho_c(L)]/\rho_c(L))$ shown in figure 4.

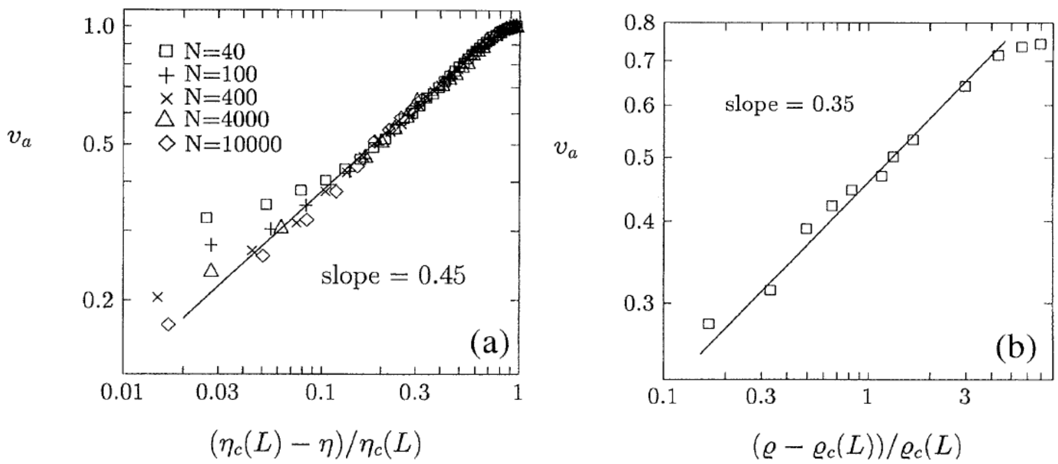

Fig. 4 (a) dependence of ln v_a on $\ln([\eta_c(L)-\eta]/\eta_c(L))$

(b) dependence of ln v_a on $\ln([\rho-\rho_c(L)]/\rho_c(L))$

Gregoire *et al.* [Gregoire *et al.*, 2003] proposed a minimal model in 2003 extending Vicsek's model by taking the possible cohesion of particles into account.

2.2 Noise-induced transition

Erdmann *et al.* [Erdmann, 2005] proposed a model with an attractive parabolic attracting pair potential between self-driven particles. Noise effect was especially investigated on the systems. The motion of N particles was given by the following equation:

$$\dot{\vec{r}}_i = \vec{v}_i \tag{5}$$

$$\dot{\vec{v}}_i = \vec{F}_i - \frac{a}{N}\sum_{j=1}^{N}(\vec{r}_i-\vec{r}_j)+\xi_i(t), \tag{6}$$

where $\vec{F}_i$ is chosen in the form of $\vec{F}_i=(1-\vec{v}_i^2)\vec{v}_i$ depending on the velocity of particles, and ξ_i is stochastic white forces with strength D independent for different particles: $<\xi_i(t)>=0$, $<\xi_i(t)\xi_j(t')>=2D\delta(t-t')\delta_{ij}$. In simulation, all particles had identical positions as well as velocities at t = 0, and noise was introduced at time t = 30. Since the swarm of particles was not isotropic, mean-square dispersion, parallel and orthogonal to the direction of its instantaneous mean velocity $\vec{V}=(1/N)\sum_i v_i(t)$, was monitored:

$$S_{\parallel}(t) = \frac{1}{NV^2(t)} \sum_{i=1}^{N} \left\{ \left[\vec{r}_i(t) - \vec{R}(t) \right] \cdot \vec{V}(t) \right\}^2, \tag{7}$$

$$S_{\perp}(t) = \frac{1}{NV^2(t)} \sum_{i=1}^{N} \left\{ \left[\vec{r}_i(t) - \vec{R}(t) \right] \times \vec{V}(t) \right\}^2, \tag{8}$$

where $\vec{R}$ is the center of mass of the cloud of particles. Figure 5 demonstrates dependence of the mean velocity of swarm on the noise intensity *D*. A sharp transition was found at 0.067 < D < 0.070, where $|\vec{V}|$ dropped dramatically to a small number. Figure 6 shows the behavior of longitudinal and transverse dispersions as noise increases. It indicates $S_{\perp}(t) \gg S_{\parallel}(t)$ until the intensity of noise *D* approaches its critical point, which means the swarm of particles is strongly squeezed in the direction of mean velocity, and after the transition the longitudinal dispersion approaches the transverse dispersion for a strong noise. Figure 6 gives the sequential snapshots showing how translational motion is transferred to rotational mode.

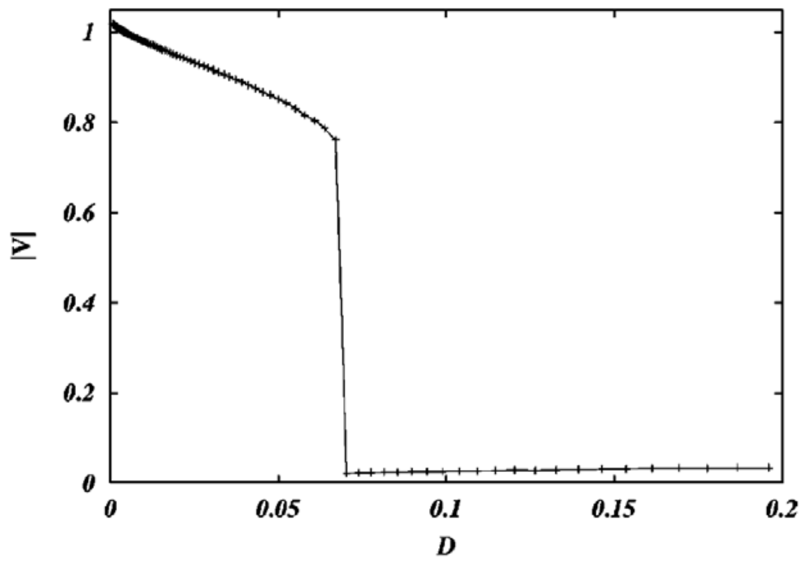

Fig. 5 Mean velocity of swarm as a function of noise intensity.

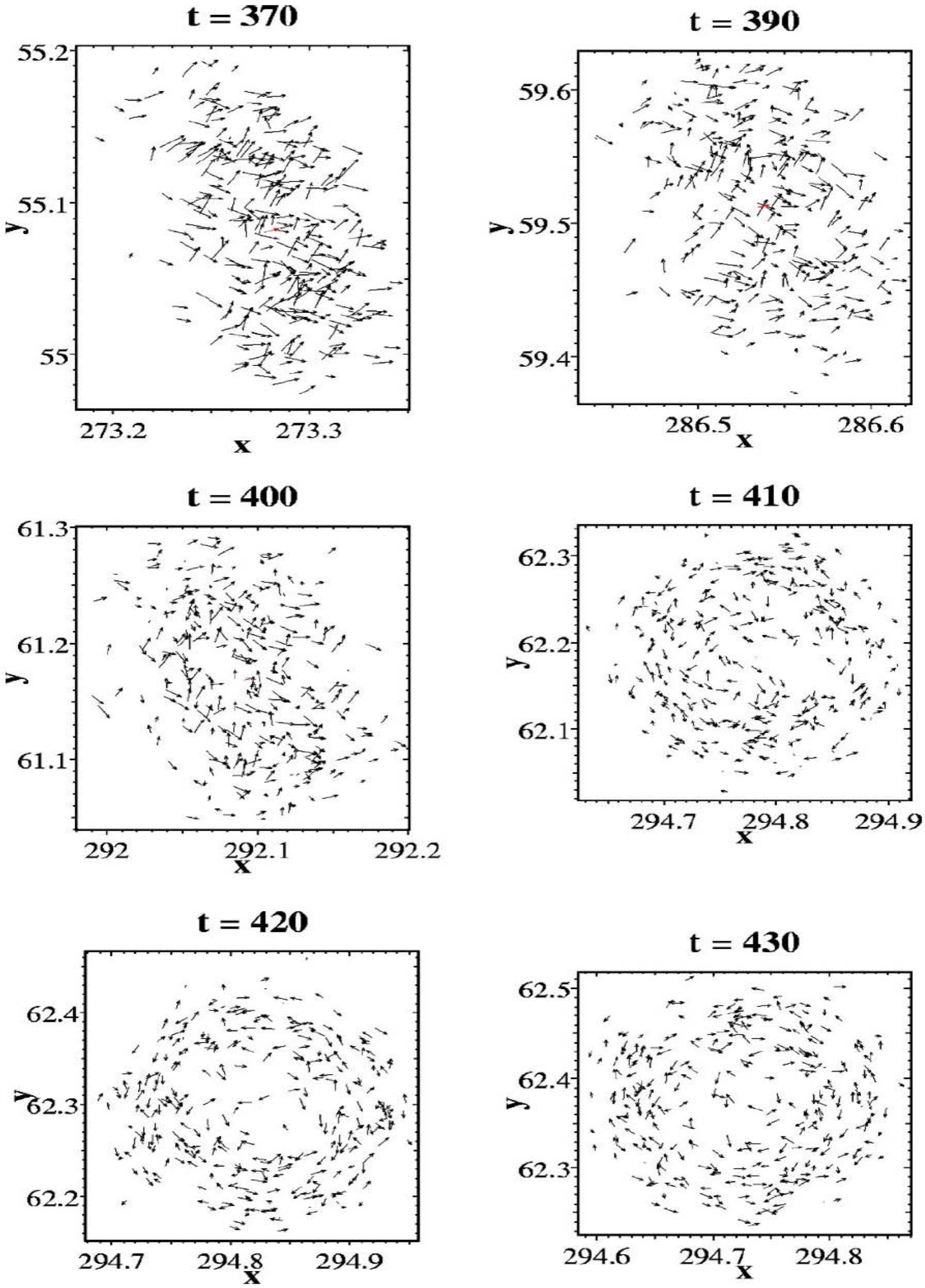

Fig. 6 Time snapshots of a swarm with noise intensity $D = 0.070$

3. Experiments

Although there are a huge number of examples of collective motion observed in nature, only a few experiments were carried out in the laboratory conditions. Becco *et al.* [Becco, 2006] presented such an experiment on fish schools by tracking the motion of every young tilapia fish. A thin container (40cm × 30cm × 2cm) with water was used, thus the motion of fish could be considered as in two dimensions. The container was illustrated by a homogeneous light source and the motion of fish was recorded by a CCD camera below the container. Figure 7 shows two typical trajectories of all fish for different fish densities (fish per m^2).

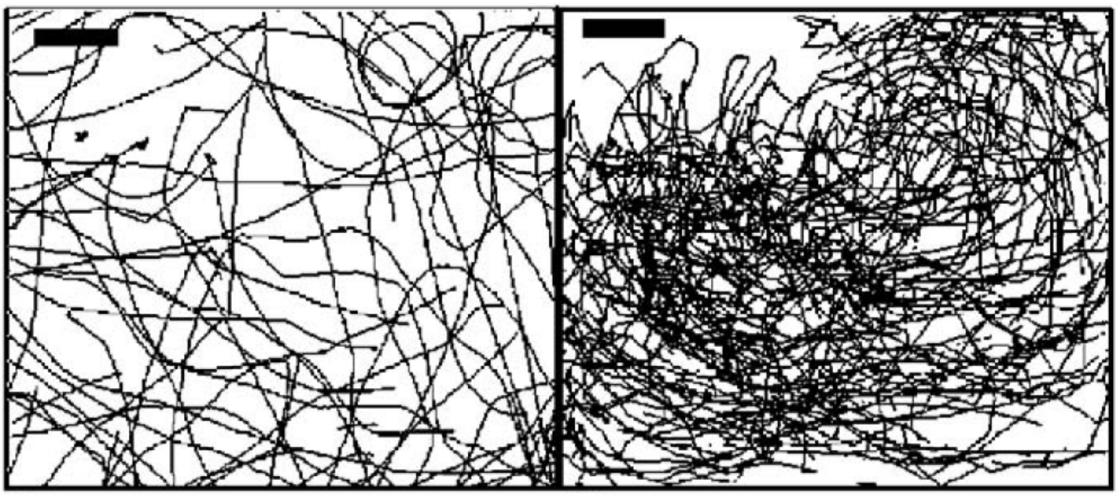

Fig. 7 Left: the trajectories of 20 fish with fish density 350 f/m^2. Right: the trajectories of 20 fish with fish density 905 f/m^2

The nearest neighbor distance *D* was computed and found to be distributed obeying a lognormal law:

$$P(D) = \frac{1}{DS\sqrt{2\pi}} \exp\left(-\frac{\left(\log(D) - \mu\right)^2}{2S^2} \right), \tag{9}$$

where μ is a scale parameter and *S* is the shape parameter. Then the position of the maximum in the distribution D_1 was calculated. The dependence of D_1 on the fish density ρ is illustrated in figure 8, showing a sharp transition at a critical density ρ_c. The behavior of $D_1(\rho)$ was fitted with a empirical law:

$$D_1 = \begin{cases} a_1\left(\rho_c - \rho\right)^\alpha + D_{1,\infty} & when\ \rho < \rho_c \\ D_{1,\infty} & when\ \rho \geq \rho_c \end{cases}, \tag{10}$$

where a_1 is a fitting parameter. The fitting gives ρ_c =527±126 f / m^2 , $D_{1,\infty}$ = 1.21±0.17 cm and exponent α = 0.7±0.3. Correlations between different fish velocities were also measured. The correlation between speeds of neighboring fish was found to be very weak, but the correlation between relative orientations of neighboring fish was strong. Figure 9 shows the distribution of the relative orientation θ_{nn} defined as the relative angle between velocities of neighboring fish. An exponential fit was used to find the width σ of the distribution. Then the cooperativeness was measured by the inverse of the width σ^{-1} and plotted in figure 10 as a function of fish density. Another empirical law was applied to fit the curve:

$$\sigma^{-1}(\rho) = \begin{cases} b_2 & when\ \rho < \rho_c \\ a_3(\rho - \rho_c)^\beta & when\ \rho \geq \rho_c \end{cases}, \tag{11}$$

where a_3 and b_2 are fitting parameters. $\rho_c = 472 \pm 38$ f/m^2 and $\beta = 0.7 \pm 0.3$ were obtained by fitting.

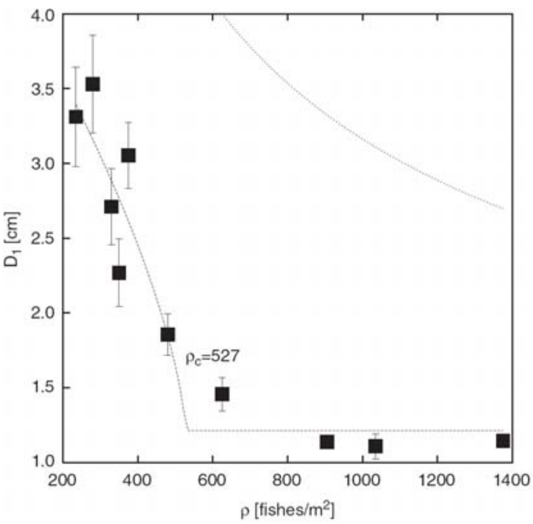

Fig. 8 The most probable interdistance D_1 as a function of fish density.

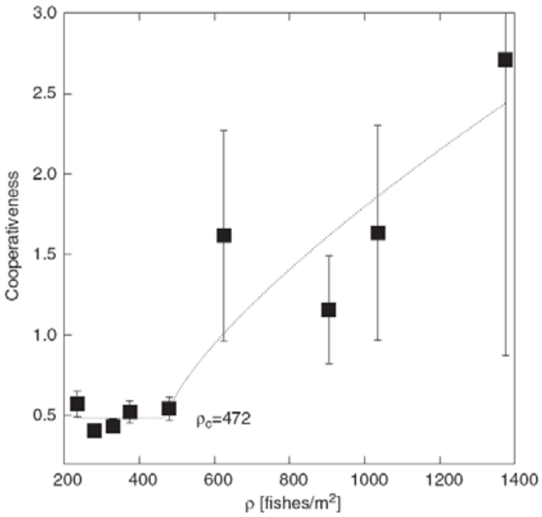

Fig. 9 Dependence of cooperativeness on fish density.

B. Szabo *et al*. [B. Szabo *et al*.,2006] presented a better controlled experiment in 2006 using tissue cells. A computer-controlled time-lapse microscope was used to monitor the motion of cells, which is presented in figure 10 for three different densities. In high density case, cells show ordered motion. The order parameter was chosen to be time average of the sum of the normalized velocities divided by the number N of cells measured:

$$\bar{V} = \left\langle \frac{1}{N} \left| \sum_{i=1}^{N} \frac{\vec{v}_i(t_k)}{|\vec{v}_i(t_k)|} \right| \right\rangle_{t_k}, \tag{12}$$

where t_k is the time elapsed. Figure 11 shows the order parameter as a function of normalized cell density indicating a sharp phase transition occurs as the normalized cell density increases. This experiment was also interpreted by the model proposed by

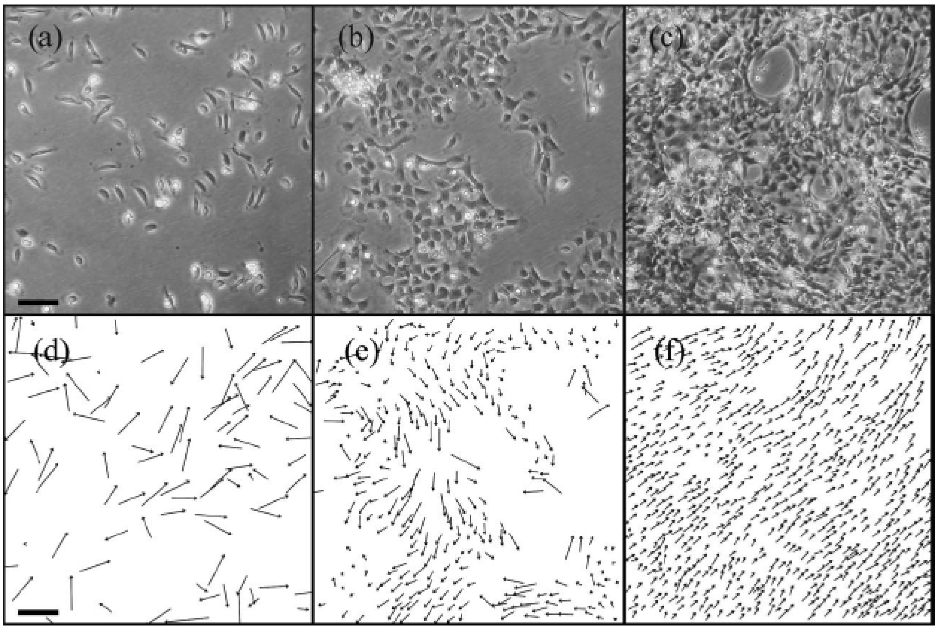

Fig. 10 Motion of cells for three different densities. (a) 1.8 cells/100×100μm^2. (b) 5.3 cells/100×100μm^2. (c) 14.7 cells/100×100μm^2.

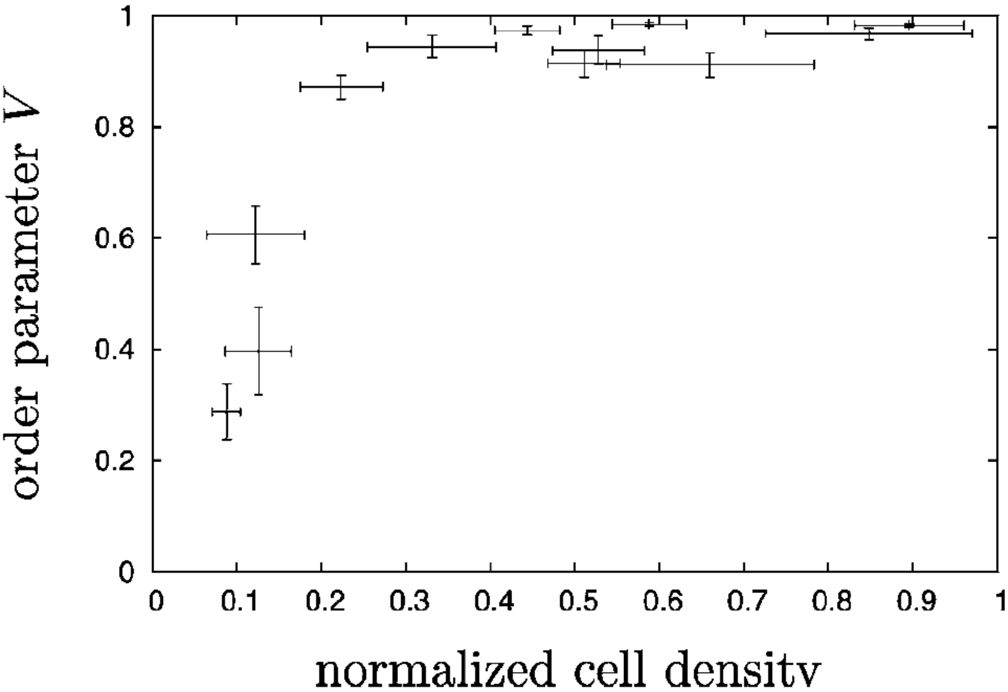

Fig. 11 Dependence of order parameter on normalized cell density.

4. Conclusion

Several models were established for collective motion of organisms. Computational simulations in these models were also performed, which gave the theoretical evidence of collective behavior, and were used to interpret experimental observations. However, the lack of experiments under laboratory conditions limits a better understanding of this vital phenomenon. How to perform better experiments which can be analyzed quantitatively is a key point.

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