One-dimensional Quantum Wires

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May 5, 2008

Abstract

In this paper we investigate the properties of a one-dimensional quantum wire of interacting electrons in the Wigner crystal limit. Recent theoretical work has explored some of the classical and quantum phase transitions associated with this system. For example, there exists a critical density above which the system can lower its energy by forming a quasi-one-dimensional zig-zag chain. We also discuss the potential relevance of such behavior to the so-called 0.7-structure.

1 Quantum Wires

1.1 Properties

A conductor will show ohmic behavior as long as its dimensions are much larger than (1) the electron de Brogilie wavelength, (2) the mean free path, and (3) the phase-relaxation length [1]. We will be exclusively interested in the first possibility. *Quantum wires* are mesoscopic devices which are good conductors in one direction but quantization in the transverse direction leads to quantized values of the conductance.

It follows from very general arguments (see [1]) that the conductance per transverse mode per electron is equal to e^2/h. Hence, the conductance of a quantum wire is $2e^2/h \times M$ where M is the number of modes in the energy range $\mu_1 < E < \mu_2$ and μ_1 and μ_2 are the chemical potentials on either side of the conductor. The factor of 2 accounts for the two possible spins of the electron.

1.2 0.7 Structure

While this simple prediction of conductance of a quantum wire is usually quite accurate at low temperatures, quantum wires generally show a 'shoulder' at $0.7 \times 2e^2/h$ as the temperature increases (see figure 1) [9]. This behavior is exhibited in a wide variety of quantum wires and there is broad consensus that the origin of this anomaly is the special nature of electronic interactions in one dimension. Spin is believed to play an important role in these effect: the 0.7-structure evolves into the spin-polarized plateau (at a conductance of $0.5 \times 2e^2/h$) with increasing magnetic field. However, the failure of the Luttinger liquid model which accurately accounts for the properties of many strongly interaction 1-d electrons suggests that these electrons are beginning to 'feel' the transverse direction.

Although a quantitative explanation of the 0.7-structure remains elusive, recent theoretical work has investigated both the role played by dimensionality and spin in quantum wires. The model we investigat this this paper provides compelling evidence for the nature of the interactions responsible for the anomaly. We find that that the Wigner crystal limit in quantum wires (even at zero temperatures) possesses very rich behavior.

2 Wigner crystal

The model we investigate consists of charged particles at zero temperature in a wire confined in the transverse direction by a potential of the form $\frac{1}{2}m\Omega^2 y^2$. A *Wigner crystal* is a solid phase of electrons in which they are (at least classically)

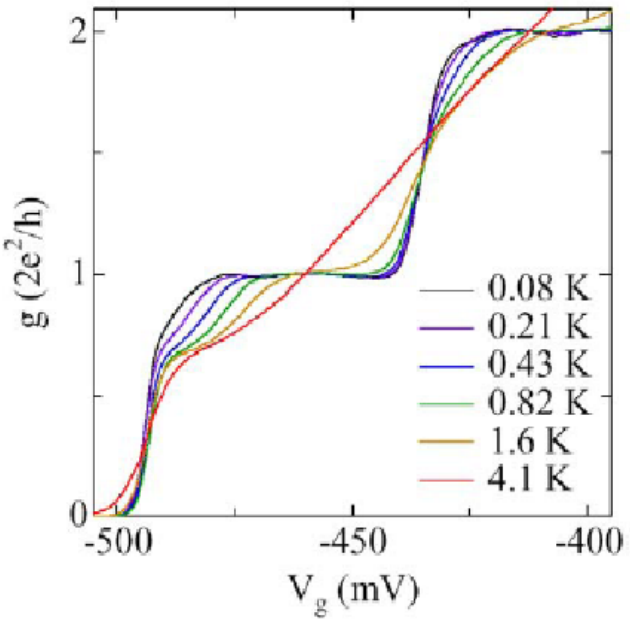

Figure 1: Conductivity as a function of bias voltage for several temperatures in GaAs wires from [9]. Note the emergence of the '0.7 shoulder' as temperature is increased.

pinned to a local minimum of the potential. For electrons interacting via a Coulomb interaction, the kinetic energy $\sim \hbar n_e^2/m$ goes to zero faster than the potential energy $\sim e^2 n_e/\epsilon$ as n_e vanishes. Hence the arrangement of the electrons becomes important becomes important in the dilute limit. In [7], the classical configurations of a such a system (of spinless electrons) was considered. In particular, they considered an interaction between electrons of the form

$$V_{int} = \frac{e^2}{2\epsilon} \sum_{k \neq l} \frac{e^{-\kappa |\mathbf{r}_k - \mathbf{r}_l|}}{|\mathbf{r}_k - \mathbf{r}_l|}, \tag{1}$$

where ϵ is the dielectric constant of the material (the addition of which allows one to consider the effect that the range of the interaction has on crystalline order). [7] investigated the surprisingly rich phase structure of this system. At very low density, the confining potential pins all the particles to the bottom of the well. Above a critical linear density the interaction potential overcomes the confining potential and the electrons split into two rows (see figure 2). This can be visualized as a kind of buckling transition. This transition is continuous in the sense that the distance between the rows vanishes at the transition. As the density is further increased, more rows appear and form increasingly complex geometric structures. Interestingly, only the first bifurcation is continuous. Figure 3 plots the lateral displacement of the rows as a function of density.

Of course, in one dimension quantum fluctuations destroy any such long-range order. However, it has been argued that the above classical predictions are robust. Specifically, Schulz ([10]) considers the fate of various correlations of a one-dimensional gas interacting with long range Coulomb forces when quantum effects are included. For example, he finds the density-density correlation

$$\langle \rho(x)\rho(0) \rangle = A_1 \cos(2k_F x) \exp(-c_2 \sqrt{\ln x}) + A_1 \cos(4k_F x) \exp(-4c_2 \sqrt{\ln x}) ... \tag{2}$$

This result is significant in that the correlations decay more slowly than any power law. For electrons in one dimension at $T = 0$ we have $n = 2k_F/\pi$. Therefore the $4k_F$ oscillation has a period that corresponds to the interparticle spacing. Indeed, Schulz predicts that this term would produce strong quasi-Bragg peaks in a scattering experiment. The *quasi* emphasizes that there is no long range order, rather quasi-long-range order in the sense that equation 2 makes precise.

3 Spin structure and the Heisenberg model

3.1 Motivation

The evidence that the 0.7-structure may involve novel spin structure has inspired an investigation of the spin properties of the quasi-one-dimensional quan-

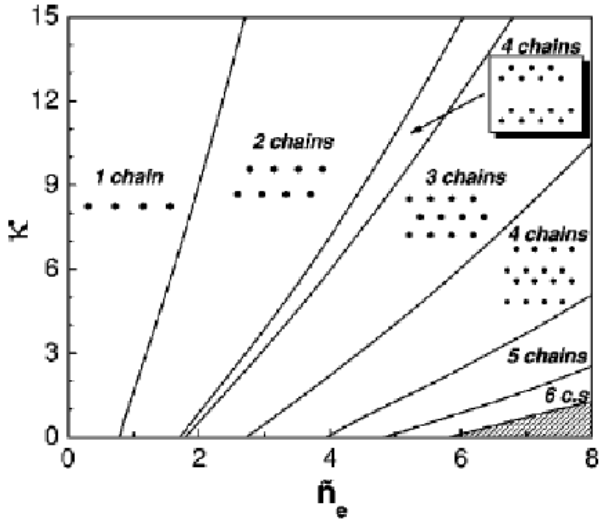

Figure 2: Zero temperature phase diagram of a 1-d harmonically confined Wigner crystal. The various regions in electron density (n_e) and dielectric constant (κ) space indicate the crystal configuration with the minimum energy [7]. Note that the phase boundaries tilt to the right. This is not surprising considering that for fixed density, increasing κ decreases the effective range of the potential.

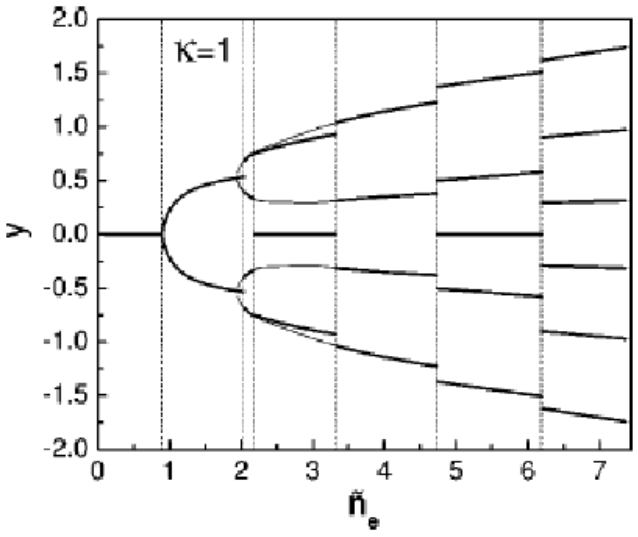

Figure 3: Transverse displacement of the most stable configuration of a chain for a given density at $\kappa = 0$. Note that the darker lines indicate the most stable configuration whereas the lighter lines show locally stable configurations that are *not* global minima. This plot is presented in [7]. The transition between 1 row and 2 rows occurs at a density $n_c = (4/7\zeta(3))^{1/3} \approx 0.780$ ([5]). All other transition must be computed numerically. Perhaps surprisingly, between the transition between 2 and 3 rows there is a small interval of electron density for which 4 rows is the most stable configuration ($n_e \approx 2.1$).

tum wire we considered in §2 [8]. For a one-dimensional Wigner crystal, the quantum mechanical exchange of neighboring electrons gives rise to an interaction which is well described by the Heisenberg model $\sum_i J_1 \mathbf{S}_i \cdot \mathbf{S}_{i+1}$. Now, for a sufficiently 'buckled' Wigner crystal, next-to-nearest neighbor interactions become important and can be modeled by the Hamiltonian

$$H = \sum_i \left(J_1 \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \mathbf{S}_i \cdot \mathbf{S}_{i+2} \right). \tag{3}$$

As pointed out in [8], the buckled phase also promotes cyclic exchanges of electrons. The case of 3-ring cyclic processes can be modeled by equation 3 through a modification of the constants J_1 and J_2. Such processes introduce complicated effects (such as frustration) and lead to highly nontrivial behavior. Additionally, whether a cyclic process involves an even or an odd number of electrons dictates whether it will be antiferromagnetic or ferromagnetic (respectively).

Once the relevant Hamiltonian is determined, how are the exchange constants calculated once the relevant processes are identified? A widely used method is the so-called instanton method. In [11], Roger presents a WKB-like approach which can calculate the the exchange constants by calculating the Euclidean time path integral over the classical exchange trajectories. These exchange constants have the form $J_\ell \sim e^{-\nu_\ell}$, where the exponent ν is a function of the geometry of the crystal. Estimates of these exchange constants presented in [8] show that in the zigzag transition regions of positive and negative J_1 and J_2 become accessible. It's clear from figure 4 that the behavior of a quasi 1-d wire can be quite complicated. As the density is changed, the relative importance of different processes changes.

3.2 Phase diagram of Heisenberg spin chain

We now discuss some of the intricacies of the (J_1, J_2) phase diagram of the Heisenberg model. For $J_2 < 0$, the interaction between nearest neighbors simply reinforces either the ferromagnetism or anti-ferromagnetism that is controlled by the first term. Hence, for $J_2 < 0$ the model is ferromagnetic or antiferromagnetic depending on whether J_1 is negative or positive, respectively. The story becomes more interesting for $J_2 > 0$ since this term will frustrate both ferromagnetic and antiferromagnetic ordering. Indeed, for $J_2 > 0.38|J_1|$ the ground state is dimerized: spin singlets form between nearest neighbors. Since a given spin can pair with an electron either to its left or right, this ground state represents a spontaneously broken Z_2 symmetry.

Now, Chubukov suggested in [13] that for $J_1 < 0$ there exists the possibility of another state (known as the chiral-biaxial-nematic phase). Chubukov argued that perhaps there exists a broken phase that is a spin triplet rather than spin singlet. Indeed, this is an exact ground state for an anisotropic version of

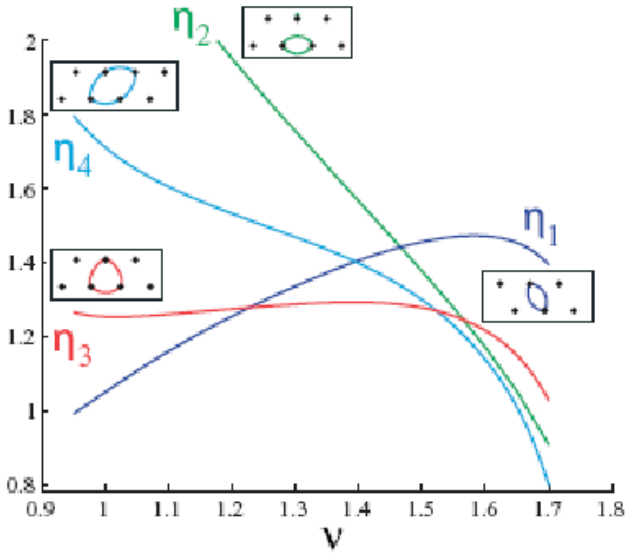

Figure 4: Exchange exponents as a function of linear electron density. This gives a sense of the competition and importance of various processes as zig-zag structure forms. (from [8]).

equation 3. Because S_z can take three different values, such a phase would be equivalent to a $S = 1$ XY spin-nematic phase (hence the fancy sounding name). Chubukov predicted that such a phase would exist for $J_1 < 0$ and $-0.25 < J_2/J_1 < -0.38$ (this corresponds to the lightly shaded wedge in figure 5). Although a spin triplet dimer does seem a plausible intermediate between the ferromagnetic and spin singlet phase, the exact nature of this phase remains controversial.

Of course, for sufficient densities, higher order interactions become relevant. Significantly, the inclusion of $S_i \cdot S_{i+3}$ and $S_i \cdot S_{i+4}$ terms can lead to regions of phase space with macroscopic spin polarization. These effects have been studied numerically in a number of papers including [8]. [14] investigated a similar Hamiltonian and proved the existence of such spin polarized states rigorously.

4 Excitations

A quasi-one-dimensional Wigner crystal at *any* density will possess a long wavelength plasmon mode with an acoustic spectrum [5]. For densities above the first zigzag transition there is also soft mode with displacements $\delta y_k = (-1)^k \varphi$.

These transverse modes are characterized by an action of the form

$$S \sim \int d\tau \; dx \left[(\partial_\tau \varphi)^2 + (\partial_x \varphi)^2 - \delta\nu\varphi^2 + \varphi^4 \right], \tag{4}$$

where $\pm\delta\nu$ are the classical minima in the y-direction in the zigzag phase. It's satisfying that the potential associated with the zigzag transition is exactly what is predicted by *phenomenological Landau theory* [12]. Although the analysis is somewhat involved, the processes of tunneling and nearest-neighbor interactions in this potential can be modeled quantum mechanically. It is found that there is a gap (Δ) that depends linearly on the energy mismatch between these two processes.

[5] points out a very interesting connection between this system and the Ising model. It turns out there is (very generally) a mapping between d-dimensional quantum systems and $(d+1)$-dimensional classical models. Indeed, the action in equation 4 can be mapped to that of the 2-D Ising model. In this particular case, the gap Δ corresponds to the inverse correlation length of the Ising model. For example, the critical behavior $r_c \sim |T - T_c|^{-1}$ corresponds to the linear behavior of the gap discussed above.

5 Carbon nanotubes

Carbon nanotubes are honeycomb lattices rolled into a cylinder. They have attracted a great deal of attention in recent years because of their novel electronic and mechanical properties [2]. Indeed, low energy electronic excitations are well modelled by massless Dirac fermions. Transverse states in the nanotube correspond to quantized angular momentum states around the nanotube's circumference. The relative ease with which nearly defect-free nanotubes can be obtained experimentally has made them a very exciting material and their transport properties have been well studied. Although the 0.7-structure is traditionally associated with more conventional quantum wires like GaAs, recent work presented in [15] on nanotubes has revealed an unmistakable 0.7 signature in the conductance of carbon nanotubes.

6 Outlook

Whether or not this picture of quasi-one-dimensional Wigner crystals can provide an explanation of the 0.7-structure, the physics of the model remains intrinsically interesting. It is the potential tuneability of this system which makes it so interesting experimentally. Indeed, as pointed out before, the so-called chiral-biaxial-nematic phases nature remains controversial and very hard to model (even numerically). There is a great deal of interest in using cold

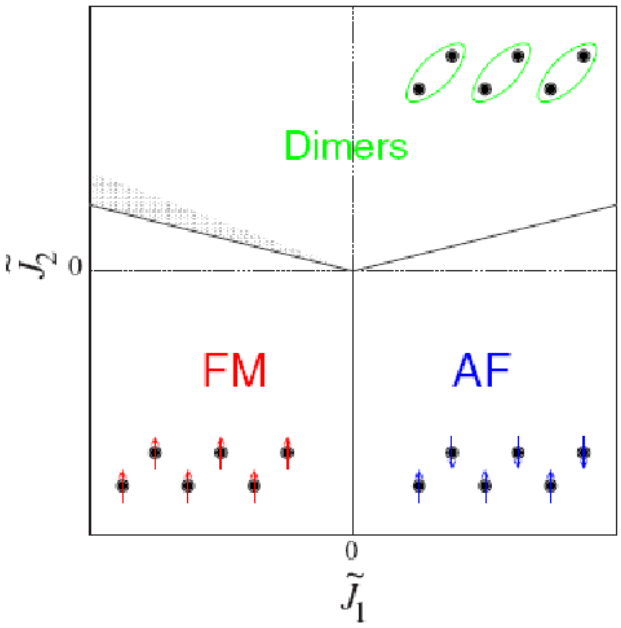

Figure 5: The phase diagram of the Heisenberg model with nearest and next-to-nearest neighbor interactions (from [8]).

atomic gases to map phase diagrams of various Hamiltonians. Recent advances in the ability to design and manipulate quantum wires could make such modeling a reality in these systems as well.

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