**Physics 504: Statistical Mechanics and Kinetic Theory**

**HOMEWORK SHEET 9 - FINAL EXAM**

Due 11am Wed May 13th 2020

*Please write this statement at the top of your solutions.*
**This exam is my own work and I did not discuss it with other students.**
**I will not redistribute this exam to others.**

**NAME:**_____

Please respect the following requests. You may not use books to help you in this exam, but you may use your notes and homework solutions. You may not confer with other students in the class about the exam. You may not make copies of the exam.

**Question 9-1.**

Electrons can be thermally excited from a metal, thus inducing a positive charge at the surface. The escaped electrons are attracted by the positive charge, and thus there is an average density distribution $n(x)$ of electrons a distance $x$ outside the metal — a so-called *space charge.*

(a) What are the boundary conditions on the electrostatic potential at infinity?

(b) Assuming that the density is sufficiently small that classical statistical mechanics is applicable, show that $n(x)/n(0) = (1 + x/L)^{-2}$, and find the length scale $L$ in terms of $n(0)$, $T$ and fundamental constants. [*Hint: in this problem, no approximation is necessary! There is an integrating factor for the equation you need to solve.*]

**Question 9-2.**

These are all short, quick questions. You should not need to do a lot of calculation, and some are just estimations.

(a) A pencil is balanced on its blunt end on a flat table. Model the pencil as a homogeneous cylindrical rod of diameter $d$, length $\ell \gg d$ and mass $m$. Guess typical values for these parameters and estimate the temperature at which the probability of the pencil falling over due to thermal fluctuations becomes of order unity. Neglect combustion!

(b) The viscosity $\eta$ of a plasma of singly-charged ions of mass $m_i$ and electrons might be expected *a priori* to depend upon temperature $T$, $m_i$, the density of ions $n$ and the electronic charge $e$. In fact, it is found experimentally that to a good approximation, $\eta$ is independent of $n$. Using this finding, find the functional form of the dependence of $\eta$ on temperature. Hint: use dimensional analysis type of reasoning.

(c) In 1908, Perrin made observations of the height distribution of small particles suspended in water at room temperature. He found that over a vertical distance of $10^{-2}$ cm, the number of particles was halved. The particles had density 1.35 gm/cm$^3$, and volume $9.8 \times 10^{-15}$ cm$^3$. Determine Boltzmann's constant.

(d) At high temperatures $T$ where the thermal energy per particle greatly exceeds the rest mass energy, the collisions of ultra-relativistic particles give rise to the formation of electron-positron pairs which come into thermal equilibrium with electromagnetic radiation through the 'chemical reaction'

$$e^+ + e^- \rightleftharpoons \gamma$$

where $\gamma$ denotes photons. The electrons and positrons are created and annihilated in pairs. Show that the number of electrons is of the form $N_e = BVT^n \int_0^\infty x^2(1+\exp(x))^{-1}dx$, and determine the constant $B$ and exponent $n$.