Physics 504: Statistical Mechanics and Kinetic Theory
HOMEWORK SHEET 4

Due 4pm Thur 26 March 2020 in the 504 box.

Please attempt these questions without looking at textbooks, if you can. If you do need to refer to my notes or textbooks, the most effective way to do this is to read the relevant section, and then try the question again without looking at the book/notes.

Question 4–1. (Don't spend too long on this question please)

Paradox: How can the chemical potential μ be negative, as it is required to be for ideal bosons? After all, it is the Gibbs free energy per particle, so surely it should be positive? In this problem, resolve this paradox qualitatively by using the equivalent definition

$$\mu \equiv \left.\frac{\partial E}{\partial N}\right|_{S,V}.$$

Here are some steps to guide you in your thinking:

(a) What happens to the entropy of a system when (*e.g.*) at or near zero temperature a particle is added to the system with no change in *energy*? Thus, with what sign of energy should one add a particle to the system in such a way that the *entropy* does not change?

(b) How are the considerations above modified for Fermi systems?

Question 4–2.

(a) By appropriately differentiating our general expression for Ξ show that in the grand canonical ensemble, for free fermions

$$\langle n_i \rangle = \frac{\lambda e^{-\beta\epsilon_i}}{1+\lambda e^{-\beta\epsilon_i}}$$

with λ being the fugacity $e^{\beta\mu}$.

(b) Calculate the Fermi energy ϵ_F for ideal fermions with spin s.

(c) Briefly explain why the $\lambda \gg 1$ limit is the appropriate one for low temperature.

(d) Using the number constraint equation and the leading asymptotic form of the function $f_{3/2}(\lambda)$ for $\lambda \gg 1$ given in lectures, for free fermions with spin s at low temperatures, calculate the explicit expression for the temperature dependence of the chemical potential to lowest non-trivial order in T and show that

$$\mu(T) = \epsilon_F[1 + AT^2 + O(T^3)]$$

where A is a coefficient that you must determine (be careful about the sign). Verify that the correction to the zero temperature value of $\mu(T)$ is indeed small as claimed in lectures.

Question 4–3.

A cylinder is separated into two compartments by a free, sliding piston. Two ideal Fermi gases are placed into the two compartments, numbered 1 and 2. The particles in compartment 1 have spin 1/2. The particles in compartment 2 have spin 3/2. All particles have the same mass.

(a) What is the equilibrium relative density of the two gases at $T = 0$?

(b) What is the equilibrium relative density of the two gases as $T \to \infty$?