

**Physics 504: Statistical Mechanics and Kinetic Theory**  
**HOMEWORK SHEET 9**

**Due Mon 30 April 2007 at 5pm NO LATE ENTRIES!**

The final will be on Fri May 4 2007 at 1.30-4.30 pm in 136 LLP. You may bring into the exam room a single regular sheet of paper on ONE SIDE of which you may write anything you believe will help you in the exam.

**Question 9–1.**

Consider a particle which lives on the sites of a one dimensional lattice. Let the site separation be  $a$  and suppose that every time interval  $\tau$ , the particle hops at random to the right or the left, with equal probability. This is a model of electron motion in a quasi-one dimensional system, for example.

- (a) Find an *exact* expression for  $P(d, N)$ , the probability that the particle is at a distance  $d$  from the origin after  $N$  jumps, using combinatorial arguments.
- (b) Using Stirling's approximation, to the accuracy that you deem necessary (you should explain and justify this choice), derive an approximation for  $P(d, N)$  valid for large  $N$ . [Stirling's formula states that  $n! = \sqrt{2\pi n} n^n \exp\{-n + 1/n + O(1/n^2)\}$ ]. Use the approximation to calculate  $\langle d^2 \rangle$ .
- (c) Construct a partial differential equation satisfied by  $P(d, N)$  in the long wavelength limit, together with appropriate initial and boundary conditions, and verify that your answer in (b) is indeed the solution.

**Question 9–2.**

A polymer chain in solution may be modelled as a random walk with step length  $l$  and total length  $L = Nl$ , where  $N$  is the number of monomers in the chain. We will sometimes find it convenient to model the chain as a continuous curve, with an arc length  $s$ , with  $0 \leq s \leq L$ .

- (a) What is the average end-to-end distance of the chain?
- (b) Write down a partial differential equation for  $G(\mathbf{r}, \mathbf{r}'; s, s')$ , the probability that the point on the chain at arc length  $s$  is at  $\mathbf{r}$  and the point on the chain at arc length  $s'$  is at  $\mathbf{r}'$ . The solution of this equation was derived in class. Hence write down  $G(\mathbf{r}, \mathbf{r}'; s, s')$ .
- (c) A valuable probe of the conformation of a system of  $N$  objects with positions  $\mathbf{r}_i$  is scattering of light, neutrons or X-rays. The cross-section is proportional to the structure function

$$S(\mathbf{k}) = \frac{1}{N} \left\langle \sum_{ij=1}^N e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right\rangle,$$

where the average is with respect to the probability distribution for the positions. Calculate the structure function for a polymer chain, and sketch the form of the result, indicating the asymptotic form of the behaviour at small and large  $k$ .

- (d) Under what circumstances would this be a good model? What features of real polymers are left out in this description? To what extent would you expect them to change the results above?