Physics 504: Statistical Mechanics and Kinetic Theory
HOMEWORK SHEET 8

Due 5pm Fri April 20 in the 504 box.

Question 8–1.

Consider a fluid system described by the grand partition function

$$\Xi(\lambda, V) = (1+\lambda)^V \left(1+\lambda^{\alpha V}\right),$$

where $\alpha > 0$ and V is the volume of the system in some units, such as the hard core radius, and λ is the fugacity. This form of Ξ is chosen to illustrate some points of principle; I do not know if there is a real system with this form of Ξ.

(a) Write down the equation of state in parametric form (i.e. in terms of λ), and sketch the graphs of p and v^{-1} versus λ, in the thermodynamic limit. Show that there is a first order transition, and find the specific volumes of the two phases at coexistence.

(b) Find the roots of $\Xi(\lambda, V) = 0$ in the complex λ plane at fixed V. Show that they converge to the point on the real λ axis at which the phase transition in (a) occured, in the thermodynamic limit.

(c) Find the equation of state in the gas phase, and notice that it exhibits no signature of the phase transition as the volume per particle is decreased beyond that where the phase transition occurs according to (a). Explain.

(d) Check that the form of the grand partition function, which we have assumed, is physically acceptable, by calculating the isothermal compressibility, and explain the significance of this calculation.

Question 8–2.

This question concerns the mean field theory for the Ising model discussed in class.

(a) Explain why the solution $M = 0$ is not physically acceptable in the mean field theory of the zero field Ising ferromagnet below T_c (perhaps by considering the behavior of the isothermal susceptibility).

(b) Calculate the isothermal susceptibility χ_T for zero external field H for $T < T_c$ and verify that the way it diverges very close to T_c is of the form

$$\chi_T \sim \frac{A_-}{T_c - T}, \quad \text{as } T \to T_c^-$$

and determine A_-.