**Physics 504: Statistical Mechanics and Kinetic Theory**
**HOMEWORK SHEET 3**

Due 5pm Mon 26 Feb 2007 in the 504 box.

*Please attempt these questions without looking at textbooks, if you can. If you do need to refer to my notes or textbooks, the most effective way to do this is to read the relevant section, and then try the question again without looking at the book/notes.*

**Question 3–1.**

(a) Let the Helmholtz free energy of a system with $N$ particles be $F = U - TS$. By considering the condition for diffusive equilibrium between two systems in contact or otherwise, show that the chemical potential is $\mu = \partial F/\partial N$.

(b) Why can we not conclude that $\mu = F/N$, whereas the conclusion that $\mu = G/N$, with $G$ being the Gibbs free energy, is valid.

**Question 3–2.**

(a) Consider a system with three energy levels, with energies $E_1$, $E_2$, $E_3$, and with degeneracies 1, 3 and 2 respectively. Write down the partition function for the system, when it is at equilibrium with temperature $T$.

(b) Write down the correctly normalised probability distribution $f(E)$ that an ideal semi-classical gas with $N$ particles has energy $E$, when it is in thermal equilibrium at temperature $T$. Sketch $f(E)$ and verify that it is sharply peaked about the energy calculated in the microcanonical ensemble. Find the form of the distribution as $N \to \infty$, and show that as this limit is taken, the distribution becomes sharper and sharper, as we discussed on general grounds in class.

**Question 3–3.**

(a) From the formula for the energy fluctuation $\Delta E$ derived in class, propose a formula for the temperature fluctuation $\Delta T$. This calculation is elementary (two lines of algebra), as long as the density is held constant. In fact, the result you will obtain is equally valid when density fluctuations are allowed also: volume and temperature fluctuations are uncorrelated.

(b) The nucleus of an atom can be modelled as a gas of nucleons with internal energy $U$ given by $U = N(k_BT)^2/\alpha$ where $\alpha = 8$ MeV. If $U$ is 81MeV, how large must $N$ be in order that the relative temperature fluctuations $\Delta T/T$ be less than 0.1? Comment on the applicability of the concept of temperature in nuclear physics.

(c) Do you think the concept of a "temperature fluctuation" makes sense in (i) the canonical distribution?; (ii) the microcanonical distribution? Briefly justify your answers. [No credit will be given for this question - I only want you to think and discuss about something that is interesting and somewhat controversial. Please see the course web site for some articles on this question.]

**Question 3–4.**

Chromium methylamine alum contains $Cr^{+++}$ ions which may exist in either of two states, both doubly degenerate. The heat capacity per ion is measured between $0.4K$ and $1.0K$, and is found to fit the curve $C = k_B(T_0/T)^2$, with $T_0 = 0.14K$. Find the energy separation of the states. [*Boltzmann's constant is* $k_B = 1.38 \times 10^{-23} JK^{-1}$.]

**Question 3–5.**

The energy levels of a quantum mechanical rigid rotator (a model for the rotational degrees of freedom of a diatomic molecule) with moment of inertia $I$ are

$$E_j = \frac{\hbar^2}{2I} j(j+1) \quad j = 0, 1, 2, 3, \ldots$$

Each level is $(2J + 1)$-fold degenerate.

(a) Write down the general expression for the partition function of this rotator.

(b) Show that at high temperatures, the expression for (a) can be approximated by an integral. What is the range of temperatures for which this is valid? In this limit, perform the integral to obtain the partition function, the internal energy and the heat capacity.

(c) Find low temperature approximations to these quantities. For what range of temperatures are your expressions valid?